Universität
des
Saarlandes
FR Informatik

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Tutorials for "Automated Reasoning II"
Exercise sheet 2

Exercise 2.1:
Use the Simplex algorithm to show that the following set of formulas is LRA-satisfiable:

$$\begin{aligned}
u &= x + y \\
v &= x - z \\
w &= z + 2y \\
u &\geq 2 \\
v &\leq 0 \\
w &\leq 4 \\
z &\geq \delta && \text{(i.e., } z > 0) \\
z &\leq 1 - \delta && \text{(i.e., } z < 1)
\end{aligned}$$

Use the variable ordering $u \prec v \prec w \prec x \prec y \prec z$.

What happens if we add the additional bound $w \leq 3$?

Exercise 2.2:
(1) Use the nondeterministic Nelson–Oppen method to show that the following formula is unsatisfiable in the combination of EUF and linear rational arithmetic:

$$\exists x, y\, (x + y \approx 0 \,\wedge\, f(x) - f(-y) \approx 1)$$

(If you choose the equations to split cleverly, the proof is quite short.)

(2) Use the deterministic Nelson–Oppen method for the same problem.

Exercise 2.3:
The conditions of Thm. 2.9 are a bit stronger than necessary. Can you think about weaker conditions that are still sufficient to prove the theorem (with almost the same proof)?

Exercise 2.4:
Find a simple example that demonstrates that the deterministic Nelson-Oppen combination procedure remains incomplete for the combination of non-linear real arithmetic and EUF if we change the calculus in such a way that not only entailed equations but also entailed negated equations are propagated.

Exercise 2.5:
Is the theory of abelian groups stably infinite? Give an explanation. (Hint: If $(G_1, +_1, 0_1)$ and $(G_2, +_2, 0_2)$ are abelian groups, then the cartesian product $(G_1 \times G_2, +, 0)$ with $(x, y) + (x', y') := (x +_1 x', y +_2 y')$ and $0 := (0_1, 0_2)$ is again an abelian group.)

Exercise 2.6:
Show that the theory described by the following set of axioms is not stably infinite.

$$\forall x\,(x * 0 \approx 0)$$
$$\forall x\,(x * 1 \approx x)$$

Bring your solution to the tutorial on May 27 and compare it with the solution that is discussed there. Your solution will not be graded.