MAX PLANCK INSTITUTE
FOR INFORMATICS

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FR Informatik

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Tutorials for "Automated Reasoning II"
Exercise sheet 1

Exercise 1.1:
Refute the following set of formulas (i) using tableaux with ground instantiation, and (ii) using free-variable tableaux:

$$\forall x\, \exists y\, P(x, y) \land P(x, x) \qquad (1)$$
$$\exists z\, \forall w\, \neg P(f(z), w) \lor \neg P(z, z) \qquad (2)$$

(If you use tableaux with free variables, use $v_1, v_2, v_3, \ldots$ as names for free variables.)

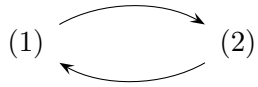
Exercise 1.2:
(a) Let $\Sigma = (\Omega, \emptyset)$ with $\Omega = \{f/2,\ g/2,\ h/1,\ k/1,\ b/0\}$. Compute the dependency pairs of the following rewrite system R over Σ:

$$f(x, h(x)) \to h(k(x)) \qquad (1)$$
$$f(h(x), y) \to g(x, g(h(x), x)) \qquad (2)$$
$$g(x, x) \to f(x, x) \qquad (3)$$
$$g(x, y) \to y \qquad (4)$$
$$h(b) \to b \qquad (5)$$

(b) Compute the approximated dependency graph for R (using cap and ren) and use the subterm criterion to show that R is terminating. If a graph is modified, depict both the original and the modified graph and indicate the strongly connected components in the graphs.

(c) The approximated dependency graph contains an edge from a dependency pair generated by rule (3) to a dependency pair generated by rule (1). Is this edge also contained in the exact dependency graph? Give an explanation.

Exercise 1.3:
(a) Find an example of a terminating rewrite system R with exactly two rewrite rules such that $\mathrm{DP}(R)$ contains exactly two dependency pairs and the dependency graph of R has the shape



Specify both R and $\mathrm{DP}(R)$.

(b) Find an example of a terminating rewrite system R with exactly two rewrite rules such that $\mathrm{DP}(R)$ contains exactly two dependency pairs and the dependency graph of R has the shape

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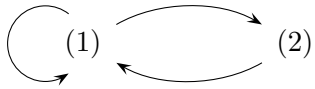

Specify both R and $\mathrm{DP}(R)$.

Bring your solution to the tutorial on April 22. and compare it with the solution that is discussed there. Your solution will not be graded.