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Tutorials for "Automated Reasoning II"
Exercise sheet 7

Exercise 7.1:
Refute the following set of clauses using AVATAR:

$$g(x,x) \approx c \ \lor \ g(x,h(x)) \approx c \ \lor \ g(h(y),y) \approx c \qquad (1)$$

$$h(h(z)) \approx h(z) \ \lor \ g(x,y) \not\approx c \qquad (2)$$

$$g(h(b),h(b)) \not\approx g(h(d),h(d)) \ \lor \ h(h(h(c))) \not\approx h(c) \qquad (3)$$

Exercise 7.2:
Let $\succ$ be an LPO with the precedence $f > g > h > b > c$; let N be the set of constrained clauses

$$f(x) \approx c \ [\![\top]\!] \qquad (1)$$

$$g(x) \approx x \ [\![x = b]\!] \qquad (2)$$

We define redundancy as on page 70 of the lecture notes: A clause C is redundant w. r. t. N, if for every confluent ground rewrite system R contained in $\succ$ every R-variable irreducible ground instance of C follows from smaller R-variable irreducible ground instances of clauses in N and smaller rules in R.

Are the following clauses redundant w. r. t. N?

$$h(f(b),x) \approx h(c,x) \ [\![\top]\!] \qquad (3)$$

$$h(g(b),c) \approx h(b,c) \ [\![\top]\!] \qquad (4)$$

$$h(g(x),x) \approx h(x,x) \ [\![x = b]\!] \qquad (5)$$

Exercise 7.3:
Refute the following set of clauses by hierarchic superposition; use linear rational arithmetic as base specification. The constants b and c are assumed to be Skolem constants of the base signature.

$$f(f(x+1)) \approx x \qquad (1)$$

$$f(b) \approx c \qquad (2)$$

$$f(c) \approx b+1 \qquad (3)$$

Bring your solution (or solution attempt) to the tutorial on July 9.