max planck institut
informatik

Universität
des
Saarlandes
FR Informatik

Uwe Waldmann **June 17, 2022**

Tutorials for "Automated Reasoning II"
Exercise sheet 6

Exercise 6.1:
Prove Lemma 3.14: For every run $N_0 \vdash N_1 \vdash N_2 \vdash \ldots$ of the superposition calculus, $Red(N_i) \subseteq Red(N_\infty)$ and $Red(N_i) \subseteq Red(N_*)$.

Exercise 6.2:
Let N be a set of equational clauses such that $\bot \notin N$. In Thm. 3.9, we have shown that whenever N is saturated up to redundancy, then every ground instance $C\theta \in G_\Sigma(N)$ is either productive or true in $R_{C\theta}$. The converse does not hold, not even for ground unit clauses: Give a (small) set of ground unit clauses N such that $\bot \notin N$ and every $C \in N$ is either productive or true in R_C, but N is not saturated up to redundancy.

Exercise 6.3:
How would you redefine the fairness of a run if saturation is defined using redundant inferences? Try to find the easiest possible definition. Reprove Lemma 3.16 for the new definitions of saturation and fairness.

Exercise 6.4:
Let $D = D' \vee t \approx t'$ and $C[u]$ be two clauses such that there is a (positive or negative) superposition inference between D and C with conclusion $C_0 = (D' \vee C[t'])\sigma$, where σ is the mgu of t and u. Suppose that $t\sigma$ occurs at least once in $C[t']\sigma$. Let C_0' be the clause that we obtain from C_0 if every occurrence of $t\sigma$ within $C[t']\sigma$ is replaced by $t'\sigma$. (As an example, consider $D = g(x) \not\approx g(y) \vee f(x,y) \approx f(y,x)$, $C = h(f(g(b),z)) \approx f(g(b),z)$, $t = f(x,y)$, $t\sigma = f(g(b),z)$, $C_0 = g(g(b)) \not\approx g(z) \vee h(f(z,g(b))) \approx f(g(b),z)$, $C_0' = g(g(b)) \not\approx g(z) \vee h(f(z,g(b))) \approx f(z,g(b))$.)

(a) C_0' is entailed by D and C_0. Why?

(b) C_0 is not redundant w.r.t. $\{D, C_0'\}$. Why?

(c) The inference that produces C_0 is redundant w.r.t. $\{D, C_0'\}$. Why?

Hint 1: Read the definitions of redundant inferences and instances of inferences really carefully. Hint 2: The ordering restrictions are an integral part of the definition of superposition inferences.

Exercise 6.5:
Find an unsatisfiable clause set consisting of two unit clauses $s \approx t$ and $u \not\approx v$ and a term ordering $\succ$ such that the only inference that neither violates the ordering restrictions of the superposition calculus nor yields a tautology is a positive superposition inference in which the left-hand side of $s \approx t$ is unified with the right-hand side of a renamed copy of $s \approx t$.

Bring your solution (or solution attempt) to the tutorial on June 24.