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Tutorials for "Automated Reasoning II"
Exercise sheet 2

Exercise 2.1:
(1) Use the nondeterministic Nelson–Oppen method to show that the following formula is unsatisfiable in the combination of EUF and linear rational arithmetic:

$$\exists x, y\,(x + y \approx 0 \,\wedge\, f(x) - f(-y) \approx 1)$$

(If you choose the equations to split cleverly, the proof is quite short.)

(2) Use the deterministic Nelson–Oppen method for the same problem.

Exercise 2.2:
Find a simple example that demonstrates that the deterministic Nelson-Oppen combination procedure is incomplete if one of the theories is not convex.

Exercise 2.3:
Let $\Sigma = (\Omega, \emptyset)$ be a signature without predicate symbols (except built-in equality). For two Σ-algebras $\mathcal{A}$ and $\mathcal{B}$, we define the product $\mathcal{A} \times \mathcal{B}$ as the Σ-algebra whose universe is the cartesian product of the universes of $\mathcal{A}$ and $\mathcal{B}$, and where $f_{\mathcal{A}\times\mathcal{B}}((a_1, b_1), \ldots, (a_n, b_n)) = \big(f_{\mathcal{A}}(a_1, \ldots, a_n), f_{\mathcal{B}}(b_1, \ldots, b_n)\big)$.

A Σ-theory $\mathcal{T}$ is called closed under products, if the product of any two models of $\mathcal{T}$ is again a model of $\mathcal{T}$.

Prove: If $\mathcal{T}$ is closed under products, then it is convex.

Exercise 2.4:
Prove: If the axioms of the Σ-theory $\mathcal{T}$ are unversally quantified equational Horn clauses (that is, clauses where all atoms are equations and at most one of the literals is positive), then $\mathcal{T}$ is convex. (You may use the previous exercise.)

Exercise 2.5:
Show that the theory described by the following set of axioms is not stably infinite.

$$\forall x\,(x * 0 \approx 0)$$
$$\forall x\,(x * 1 \approx x)$$

Bring your solution (or solution attempt) to the tutorial on May 13.