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Tutorials for "Automated Reasoning II"
Exercise sheet 4

Exercise 4.1:
Let $\Sigma = (\Omega, \emptyset)$ be a signature without predicate symbols (except built-in equality). For two Σ-algebras $\mathcal{A}$ and $\mathcal{B}$, we define the product $\mathcal{A} \times \mathcal{B}$ as the Σ-algebra whose universe is the cartesian product of the universes of $\mathcal{A}$ and $\mathcal{B}$, and where $f_{\mathcal{A}\times\mathcal{B}}((a_1, b_1), \dots, (a_n, b_n)) = \big(f_{\mathcal{A}}(a_1, \dots, a_n), f_{\mathcal{B}}(b_1, \dots, b_n)\big)$.

A Σ-theory $\mathcal{T}$ is called closed under products, if the product of any two models of $\mathcal{T}$ is again a model of $\mathcal{T}$.

Prove: If $\mathcal{T}$ is closed under products, then it is convex.

Exercise 4.2:
Prove: If the axioms of the Σ-theory $\mathcal{T}$ are unversally quantified equational Horn clauses (that is, clauses where all atoms are equations and at most one of the literals is positive), then $\mathcal{T}$ is convex. (You may use the previous exercise.)

Exercise 4.3:
What goes wrong if we combine the original DPLL procedure (including the pure literal rule) with a theory solver?

Exercise 4.4:
Use the CDCL(EUF) calculus to determine whether the following set of clauses is satisfiable or not:

$$f(a, b) \not\approx f(a', b') \qquad (1)$$

$$g(g(c)) \not\approx c \qquad (2)$$

$$g(d) \approx c \ \lor\ g(g(c)) \approx c \qquad (3)$$

$$a \approx a' \ \lor\ c \approx d \qquad (4)$$

$$b \approx b' \ \lor\ c \approx d \qquad (5)$$

Bring your solution (or solution attempt) to the tutorial on June 1.