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Tutorials for “Automated Reasoning II”
Exercise sheet 3

Exercise 3.1:
(1) Use the nondeterministic Nelson–Oppen method to show that the following formula is unsatisfiable in the combination of EUF and linear integer arithmetic:

$$\exists x, y\,(x + y \approx 0 \,\wedge\, f(x) + f(-y) \approx 1)$$

(If you choose the equations to split cleverly, the proof is quite short.)

(2) Use the deterministic Nelson–Oppen method for the same problem.

Exercise 3.2:
Prove Lemma 1.11: A first-order theory $\mathcal{T}$ is convex w. r. t. equations if and only if for every conjunction Γ of Σ-equations and non-equational Σ-literals and for all equations $x_i \approx x'_i$ $(1 \leq i \leq n)$, whenever $\mathcal{T} \models \forall\vec{x}\,(\Gamma \rightarrow x_1 \approx x'_1 \vee \ldots \vee x_n \approx x'_n)$, then there exists some index j such that $\mathcal{T} \models \forall\vec{x}\,(\Gamma \rightarrow x_j \approx x'_j)$.

Exercise 3.3:
Find a simple example that demonstrates that the deterministic Nelson-Oppen combination procedure is incomplete if one of the theories is not convex.

Exercise 3.4:
Is the theory of abelian groups stably infinite? Give an explanation.

Exercise 3.5:
Is the theory described by the following set of axioms stably infinite? Give an explanation.

$$\forall x\,(x * 0 \approx 0)$$
$$\forall x\,(x * 1 \approx x)$$

Bring your solution (or solution attempt) to the tutorial on May 25.