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**Tutorials for “Automated Reasoning II”**  
**Exercise sheet 10**

**Exercise 10.1:**

Consider the base specification  $SP = (\Sigma, \mathcal{C})$  with  $\Sigma = (\Xi, \Omega, \emptyset)$ , where  $\Xi = \{int\}$ ,  $\Omega$  contains *int*-sorted constants  $0, 1, -1, 2, -2, \dots$ , a Skolem constant  $k : \rightarrow int$ ,  $- : int \rightarrow int$ , and  $+ : int \times int \rightarrow int$ , and  $\mathcal{C}$  is the isomorphy class of  $\mathbb{Z}$  (with  $k$  interpreted by an arbitrary integer number).

We extend  $SP$  by a new sort *list*, new operator symbols  $cons : int \times list \rightarrow list$ ,  $car : list \rightarrow int$ ,  $cdr : list \rightarrow list$ ,  $empty : \rightarrow list$ , and  $a : \rightarrow list$ , and the clauses

$$car(cons(x, y)) \approx x \quad (1)$$

$$cdr(cons(x, y)) \approx y \quad (2)$$

$$cons(k, a) \approx cons(3, empty) \quad (3)$$

$$k + 5 \approx 7 \quad (4)$$

Use the hierarchic superposition calculus to show that the hierarchic specification is inconsistent.

**Exercise 10.2:**

As mentioned in the lecture, it is not necessary for the completeness of hierarchic superposition to abstract out concrete numbers. This does not hold for Skolem constants, though. Give a simple example of a hierarchic specification that can be refuted using hierarchic superposition if Skolem constants are abstracted out, but that cannot be refuted if the abstraction of Skolem constants is avoided.

**Exercise 10.3:**

Are the two terms  $b + x$  and  $y + c$  (with constants  $b, c$ ) unifiable with respect to associativity? If yes, compute a  $\mu$ CSU.

**Exercise 10.4:**

Are the two terms  $b + x$  and  $x + c$  (with constants  $b, c$ ) unifiable with respect to associativity? If yes, compute a  $\mu$ CSU.

Bring your solution (or solution attempt) to the tutorial on July 7.