Uwe Waldmann   **April 16, 2014**

Tutorials for "Automated Reasoning II"
Exercise sheet 1

Exercise 1.1:
Prove that Knuth-Bendix completion terminates and produces a convergent TRS if the input consists only of ground equations, the term ordering is total on ground terms, and simplification inferences are computed eagerly (that is, *Orient* and *Deduce* may only be applied if none of the *Simplify* rules is applicable).

Exercise 1.2:
On page 4 of the lecture notes we have sketched a flattening operation for sets of equations. Formalize it using an appropriate transition system in such a way that any two different D-equations have always different left-hand sides.

Exercise 1.3:
Let $\Sigma = (\Omega, \Pi)$ and $\Sigma' = (\Omega', \Pi')$ be signatures such that $\Omega \subseteq \Omega'$ and $\Pi \subseteq \Pi'$. If $\mathcal{A}$ is a Σ-algebra and $\mathcal{B}$ is a Σ'-algebra such that $U_{\mathcal{A}} = U_{\mathcal{B}}$, $f_{\mathcal{A}} = f_{\mathcal{B}}$ for every $f \in \Omega$ and $P_{\mathcal{A}} = P_{\mathcal{B}}$ for every $P \in \Pi$, then $\mathcal{A}$ is called the Σ-reduct of $\mathcal{B}$ (denoted by $\mathcal{A} = \mathcal{B}|_{\Sigma}$).

Prove the following two properties:

(i) Let $\Sigma = (\Omega, \Pi)$ with $c \notin \Omega$, let $\Sigma' = (\Omega \cup \{c/0\}, \Pi)$. Let N be a set of closed Σ-formulas and let t be a ground term occurring in a closed Σ-formula $F[t]_p$. Then the Σ-models of $N \cup \{F[t]_p\}$ are exactly the Σ-reducts of the Σ'-models of $N \cup \{F[c]_p\} \cup \{t \approx c\}$.

(ii) Let N be a set of closed Σ-formulas, let N' be a set of closed Σ'-formulas, and let F be a closed Σ-formula. If the Σ-models of N are exactly the Σ-reducts of Σ'-models of N', then $N \models F$ if and only if $N' \models F$.

Bring your solution (or solution attempt) to the tutorial on April 28.