MAX PLANCK INSTITUTE
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Tutorials for "Automated Reasoning"
Exercise sheet 9

Exercise 9.1:
Prove Prop. 3.14: Let $\mathcal{A}$ be a Σ-algebra, let F and G be Σ-formulas, and let $H = H[F]_p$ be a Σ-formula in which F occurs as a subformula at position p.

If $\mathrm{pol}(H, p) = 1$ and $\mathcal{A}(\beta)(F) \leq \mathcal{A}(\beta)(G)$ holds for all assignments β, then $\mathcal{A}(\gamma)(H[F]_p) \leq \mathcal{A}(\gamma)(H[G]_p)$ holds for all assignments γ.

If $\mathrm{pol}(H, p) = -1$ and $\mathcal{A}(\beta)(F) \geq \mathcal{A}(\beta)(G)$ holds for all assignments β, then $\mathcal{A}(\gamma)(H[F]_p) \leq \mathcal{A}(\gamma)(H[G]_p)$ holds for all assignments γ.

(It is sufficient if you consider the boolean connectives $\wedge$ and $\neg$; the other cases are proved analogously. Hint: You must prove both properties simultaneously; it is not possible to prove one of them individually.)

Exercise 9.2:
Challenge Problem: Prove part (ii) of Prop. 3.27: If $\sigma \leq \tau$ and $\tau \leq \sigma$, then there exist variable renamings δ and δ' (i.e., *bijective* substitutions mapping variables to variables), so that $x\sigma\delta = x\tau$ and $x\tau\delta' = x\sigma$ for every x in X. (Note: $\{x \mapsto y\}$ is *not* a bijective substitution, since $x\{x \mapsto y\} = y\{x \mapsto y\}$.)

Exercise 9.3:
Give an example of a most general unifier of $f(g(x, y))$ and $f(z)$ that is *not* idempotent.

Exercise 9.4:
Using the standard unification rules, compute an mgu of $P(g(x_1, x_1), g(x_3, h(x_4)))$ and $P(g(h(x_2), h(h(x_6))), g(h(x_5), x_5))$, if it exists.

Exercise 9.5:
Use the resolution calculus to derive the empty clause from

$$\neg P(b, f(w)) \quad (1)$$
$$P(x, f(y)) \ \vee \ Q(x, y) \quad (2)$$
$$\neg Q(b, z) \ \vee \ \neg P(z, b) \quad (3)$$
$$P(f(w), w) \quad (4)$$

Bring your solution to the tutorial on January 6 and compare it with the solution that is discussed there. If you are still unsure afterwards whether your solution is correct or not, feel free to ask the instructor after the tutorial. Your solution will not be graded.