MAX PLANCK INSTITUTE
FOR INFORMATICS

Universität
des
Saarlandes
FR Informatik

Uwe Waldmann **December 11, 2025**

Tutorials for "Automated Reasoning"
Exercise sheet 8

Exercise 8.1:
Let $\Sigma = (\Omega, \Pi)$ with $\Omega = \{c/0,\ f/1\}$ and $\Pi = \{P/4,\ Q/4\}$. Transform the Σ-formula

$$F \ = \ \exists v\, \forall x\, \forall y\, \forall z\, \neg \forall w \Big(\neg P(c, w, z, x) \wedge Q(w, y, f(x), v) \Big)$$

into clause normal form using the improved algorithm from Section 3.7. (There are no subformulas in F for which one should introduce a definition.)

Exercise 8.2:
Let $\Sigma = (\Omega, \Pi)$ with $\Omega = \{c/0,\ f/1\}$ and $\Pi = \{P/1,\ Q/1\}$. How many different Herbrand models does the formula

$$P(f(f(c))) \ \wedge \ \forall x\, (P(x) \rightarrow P(f(x))) \ \wedge \ \forall y \forall z\, (Q(y) \rightarrow Q(z))$$

have?

Exercise 8.3:
Let N be the following set of ground clauses:

$$\{\, P \vee Q,\ P \vee \neg Q,\ \neg P \vee Q,\ \neg P \vee \neg Q \,\}$$

(1) Show that $N \vdash_{Res} \bot$, that is, derive $\bot$ from N using the Resolution and the Positive Factorization rule.

(2) Why is it impossible to derive the empty clause from these clauses without using factorization?

Exercise 8.4:
Let N be the set containing the following ground clauses:

$$\begin{aligned} C_1 &= P(b) \vee \neg Q(b) \vee \neg Q(c) \\ C_2 &= P(b) \vee P(b) \vee Q(c) \\ C_3 &= P(c) \vee Q(b) \vee Q(c) \\ C_4 &= Q(b) \vee Q(c) \vee Q(c) \\ C_5 &= \neg P(b) \vee Q(c) \\ C_6 &= \neg P(c) \end{aligned}$$

Let the ordering on ground atoms be given by $P(b) \succ P(c) \succ Q(b) \succ Q(c)$.

(1) Order the clauses in N according to the associated clause ordering $\succ_C$.

(2) Compute the candidate interpretation $I_N^{\succ}$. Which clauses are productive, what do they produce, which clause is the minimal counterexample (if it exists)?

Bring your solution to the tutorial on December 16 and compare it with the solution that is discussed there. If you are still unsure afterwards whether your solution is correct or not, feel free to ask the instructor after the tutorial. Your solution will not be graded.