MAX PLANCK INSTITUTE
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Tutorials for "Automated Reasoning"
Exercise sheet 7

Exercise 7.1:
Transform the first-order formula

$$F \;=\; \forall x\, \exists y\, \exists z \Big(\big(P(z) \wedge \forall w\, Q(z,w) \big) \rightarrow \big(P(x) \wedge \neg R(x,y) \big) \Big)$$

into clause normal form.

Exercise 7.2:
Let the signature $\Sigma = (\Omega, \Pi)$ be given by $\Omega = \{+/2, s/1, 0/0\}$ and $\Pi = \emptyset$, and let

$$\begin{aligned} F_1 &= \forall x\, (x + 0 \approx x) \\ F_2 &= \forall x \forall y\, (x + s(y) \approx s(x + y)) \\ F_3 &= \forall x \forall y\, (x + y \approx y + x) \\ F_4 &= \neg \forall x \forall y\, (x + y \approx y + x). \end{aligned}$$

(1) Determine a Σ-algebra $\mathcal{A}$ with an universe of exactly two elements such that $\mathcal{A}$ is a model of F_1, F_2, F_3.

(2) Determine a Σ-algebra $\mathcal{A}$ with an universe of exactly two elements such that $\mathcal{A}$ is a model of F_1, F_2, F_4.

Exercise 7.3:
Let $\Sigma = (\Omega, \Pi)$ be a signature such that Ω contains at least one constant symbol. A Σ-algebra $\mathcal{A}$ is called *term-generated*, if every $a \in U_{\mathcal{A}}$ is term-generated. Prove that a closed prenex formula without existential quantifiers (possibly including equality) has a model if and only if it has a term-generated model.

Bring your solution to the tutorial on December 9 and compare it with the solution that is discussed there. If you are still unsure afterwards whether your solution is correct or not, feel free to ask the instructor after the tutorial. Your solution will not be graded.