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Tutorials for "Automated Reasoning"
Exercise sheet 6

Exercise 6.1:
A finite graph is a pair (V, E), where V is a finite non-empty set and $E \subseteq V \times V$. The elements of V are called vertices or nodes; the elements of E are called edges. A graph has a 3-coloring, if there exists a function $\phi : V \to \{0, 1, 2\}$ such that for every edge $(v, v') \in E$ we have $\phi(v) \neq \phi(v')$.

(a) Give a linear time translation from finite graphs (V, E) to propositional clause sets N such that (V, E) has a 3-coloring if and only if N is satisfiable and such that every model of N corresponds to a 3-coloring ϕ and vice versa.

(b) A 3-coloring is called complete, if for every pair $(c, c') \in \{0, 1, 2\} \times \{0, 1, 2\}$ with $c \neq c'$ there exists an edge $(v, v') \in E$ such that $\phi(v) = c$ and $\phi(v') = c'$ or $\phi(v) = c'$ and $\phi(v') = c$. Give a linear time translation from finite graphs (V, E) to propositional clause sets N such that (V, E) has a complete 3-coloring if and only if N is satisfiable and such that every model of N corresponds to a complete 3-coloring ϕ and vice versa.

Exercise 6.2:
Let $\Sigma = (\{b/0,\, c/0,\, d/0,\, f/1\}, \{P/1\})$. Does the formula

$$P(b) \wedge P(c) \wedge \neg P(d) \wedge \neg\exists x\, P(f(f(x)))$$

have a Σ-model whose universe has exactly two elements? Give an example of such a model or show that such a model does not exist.

Exercise 6.3:
Prove that a formula $\exists x\, F$ is satisfiable if and only if $F\{x \to b\}$ is satisfiable, where b is a constant that does not occur in F.

Exercise 6.4:
Prove Prop. 3.5: For every Σ-formula F, $\mathcal{A}(\beta)(F\sigma) = \mathcal{A}(\beta \circ \sigma)(F)$.
(It is sufficient if you prove the property for atomic formulas $P(s_1, \ldots, s_n)$, disjunctions $F \vee G$, and universally quantified formulas $\forall x\, F$; the other cases are proved similarly.)

Bring your solution to the tutorial on December 2 and compare it with the solution that is discussed there. If you are still unsure afterwards whether your solution is correct or not, feel free to ask the instructor after the tutorial. Your solution will not be graded.