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Tutorials for “Automated Reasoning”
Exercise sheet 5

Exercise 5.1:

Let N be the following set of propositional clauses:

$$\neg P \vee Q \vee \neg R \quad (1)$$

$$\neg P \vee T \vee U \vee \neg V \quad (2)$$

$$\neg P \vee Q \vee \neg T \vee \neg U \vee V \quad (3)$$

$$P \vee \neg Q \quad (4)$$

$$\neg R \vee \neg T \quad (5)$$

$$\neg R \vee U \quad (6)$$

$$P \vee \neg S \vee U \vee V \quad (7)$$

$$R \vee \neg S \quad (8)$$

$$R \vee \neg T \vee \neg V \quad (9)$$

$$S \vee \neg T \vee \neg U \vee V \quad (10)$$

$$T \vee \neg U \quad (11)$$

$$S \vee T \vee U \vee \neg V \quad (12)$$

$$U \vee V \quad (13)$$

Use the CDCL procedure to check whether N is satisfiable or not; if it is satisfiable, give a model. Use a reasonable strategy. If you use the *Decide* rule, use the largest undefined negative literal according to the ordering $\neg P > \neg Q > \neg R > \neg S > \neg T > \neg U > \neg V$. If you use the *Backjump* rule, determine a suitable backjump clause using the 1UIP method and use the best possible successor state for that backjump clause.

Exercise 5.2:

A Horn clause is a clause with at most one positive literal. Let N be a finite set of propositional Horn clauses and let $\varepsilon \parallel N \Rightarrow_{\text{CDCL}}^* M \parallel N$ be a CDCL derivation in which only the “Unit Propagate” rule is used. Prove: If neither “Unit Propagate” nor “Fail” can be applied to $M \parallel N$, then N is satisfiable.

Exercise 5.3:

The “Purity deletion” rule explained in the *Inprocessing* section is subsumed by other inprocessing rules. By which one(s)? Why?

Exercise 5.4:

Prove that the “RAT elimination” rule explained in the *Inprocessing* section is satisfiability-preserving:

C is called an *asymmetric tautology* w.r.t. N , if its negation can be refuted by unit propagation using clauses in N .

We say that C has the *RAT property* w.r.t. N , if it is an asymmetric tautology w.r.t. N , or if there is a literal L in C such that $C = C' \vee L$ and all clauses $D' \vee C'$ for $D' \vee \bar{L} \in N$ are asymmetric tautologies w.r.t. N .

Assume that C has the RAT property w.r.t. N . Show that $N \cup \{C\}$ is satisfiable if and only if N is satisfiable.

Bring your solution to the tutorial on November 25 and compare it with the solution that is discussed there. If you are still unsure afterwards whether your solution is correct or not, feel free to ask the instructor after the tutorial. Your solution will not be graded.