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Tutorials for "Automated Reasoning"
Exercise sheet 4

Exercise 4.1:
Let F be the propositional formula $((P \rightarrow \neg Q) \wedge R) \rightarrow (\neg P \wedge Q)$. Introduce a new variable for the subformula $(P \rightarrow \neg Q)$ and convert F into a CNF formula.

Exercise 4.2:
A partial Π-valuation $\mathcal{A}$ under which all clauses of a clause set N are true is called a partial Π-model of N.

Do the following clause sets over $\Pi = \{P, Q, R\}$ have partial Π-models that are not total Π-models (that is, models in the sense of Sect. 2.3)? If yes, give such a partial Π-model.

(1)
$$\begin{array}{ccccc} P & & & & \\ \neg P & \vee & Q & & \\ \neg P & \vee & \neg Q & \vee & \neg R \end{array}$$

(2)
$$\begin{array}{ccccc} P & & & & \\ \neg P & \vee & Q & & \\ & & \neg Q & \vee & R \\ \neg P & \vee & \neg Q & \vee & \neg R \end{array}$$

(3)
$$\begin{array}{ccccc} P & & & \vee & R \\ \neg P & \vee & Q & \vee & \neg R \\ & & \neg Q & \vee & \neg R \end{array}$$

(4)
$$\begin{array}{ccccc} \neg P & \vee & Q & & \\ & & \neg Q & \vee & R \\ P & & & \vee & \neg R \end{array}$$

Exercise 4.3:
Let F be a propositional formula and let C be a propositional clause. Prove: If every propositional variable that occurs in F occurs also in C, and if there exists a valuation $\mathcal{A}$ such that both F and C are false under $\mathcal{A}$, then $F \models C$.

Exercise 4.4:
Prove or refute: If $N = \{C_1, \dots, C_n\}$ is a finite set of propositional clauses without duplicated literals or complementary literals, and if for every $i \in \{1, \dots, n\}$ the clause C_i has exactly i literals, then N is satisfiable.

Bring your solution to the tutorial on November 18 and compare it with the solution that is discussed there. If you are still unsure afterwards whether your solution is correct or not, feel free to ask the instructor after the tutorial. Your solution will not be graded.