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Tutorials for "Automated Reasoning"
Exercise sheet 3

Exercise 3.1:
Prove that the multiset extension $\succ_{\text{mul}}$ of a total ordering $\succ$ is total.

Exercise 3.2:
Which of the following propositional formulas are valid? Which are satisfiable? Which are unsatisfiable?

(1) $\neg P$

(2) $P \to \bot$

(3) $\bot \to P$

(4) $(P \vee Q) \to P$

(5) $P \to (Q \to P)$

(6) $Q \to \neg Q$

(7) $Q \wedge \neg Q$

(8) $\neg(\neg P \wedge \neg\neg P)$

Exercise 3.3:
Let F, G, and H be propositional formulas. Prove or refute:

(1) If F is satisfiable and G is satisfiable, then $F \wedge G$ is satisfiable.

(2) If $F \models G$ holds and F is satisfiable, then G is satisfiable.

(3) If F is satisfiable and $F \to G$ is satisfiable, then G is satisfiable.

(4) If $F \vee G$ is valid, then F is valid or G is satisfiable.

(5) If $F \models G$ and $G \models H$, then $F \models H$.

(6) If $F \vee H[F]_p$ is valid, then $F \vee H[\bot]_p$ is valid.

(7) If $H[F]_p$ is valid and $H[G]_p$ is valid, then $H[F \vee G]_p$ is valid.

(8) If $H[F \wedge G]_p$ is valid, then $H[F]_p$ and $H[G]_p$ are valid.

Exercise 3.4:
Let $P \in \Pi$ be a propositional variable. For every Π-formula F let $\mathrm{np}(F)$ be the formula that one obtains if every occurrence of P is replaced by $\neg P$. For instance,

$$\mathrm{np}(\neg P \vee (Q \wedge P)) = \neg\neg P \vee (Q \wedge \neg P).$$

Prove: If a formula F is satisfiable, then $\mathrm{np}(F)$ is satisfiable. (It suffices it you consider the case that $\wedge$, $\vee$, and $\neg$ are the only boolean connectives in F; the other connectives $\rightarrow$ and $\leftrightarrow$ are handled analogously.)

Bring your solution to the tutorial on November 11 and compare it with the solution that is discussed there. If you are still unsure afterwards whether your solution is correct or not, feel free to ask the instructor after the tutorial. Your solution will not be graded.