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Tutorials for “Automated Reasoning”
Exercise sheet 13

Exercise 13.1:

Let E be the following set of equations over $\Sigma = (\{f/1, g/1, h/2, b/0\}, \emptyset)$.

$$f(h(b, x)) \approx h(f(b), f(x)) \quad (1)$$

$$g(g(x)) \approx f(g(x)) \quad (2)$$

$$g(h(x, x)) \approx h(g(x), x) \quad (3)$$

It is possible to orient the equations in E using an appropriate LPO, so that there are no critical pairs between the resulting rules. Give the precedence of the LPO and explain how the equations are oriented.

Exercise 13.2:

Decide for each of the following statements whether they are true or false:

- (1) $f(g(x)) \succ f(x)$ in every simplification ordering \succ .
- (2) $f(f(x)) \succ f(y)$ in every simplification ordering \succ .
- (3) If \succ is an LPO, then $f(x) \succ g(x)$ implies $f(x) \succ g(g(x))$.
- (4) If \succ is a KBO, then $f(x) \succ g(x)$ implies $f(x) \succ g(g(x))$.
- (5) If \succ is an LPO, then $h(f(x), y, y) \succ h(x, z, z)$.
- (6) If \succ is a KBO, then $h(f(x), f(y), z) \succ h(x, f(z), y)$.
- (7) There is a reduction ordering \succ such that $f(x) \succ g(f(x))$.
- (8) There is a reduction ordering \succ such that $f(f(x)) \succ f(g(f(x)))$.

Exercise 13.3:

Suppose that we replace condition (1) in the definition of the Knuth-Bendix ordering by

$$(1') \text{ var}(t) \subseteq \text{var}(s) \text{ and } w(s) > w(t).$$

Show that the resulting relation is *not* a simplification ordering.

Exercise 13.4:

Let $\Sigma = (\{f/1, g/1, h/1, b/0, c/0\}, \{P/2, Q/1, R/2\})$; let N be the following set of clauses over Σ :

$$P(f(x), x) \vee P(c, x) \vee R(g(x), x) \quad (1)$$

$$\neg P(y, f(y)) \quad (2)$$

$$\neg P(y, c) \vee \neg P(z, h(y)) \vee Q(z) \quad (3)$$

$$Q(b) \vee Q(x) \vee \neg R(g(x), x) \quad (4)$$

$$R(g(c), y) \quad (5)$$

(a) Suppose that the atom ordering \succ is a lexicographic path ordering with the precedence $P > Q > R > f > g > h > b > c$ and that the selection function sel selects no literals. Compute all $\text{Res}_{sel}^>$ inferences between the clauses (1)–(5). (Do not compute inferences between derived clauses. Do not compute any inferences that violate the restrictions of the calculus.)

(b) One of the conclusions of the inferences computed in part (a) is (relatively obviously) redundant w.r.t. N . Which one? Why?

Exercise 13.5:

Let E be the following set of equations over $\Sigma = (\{f/2, g/1, b/0, c/0\}, \emptyset)$.

$$f(x, x) \approx f(x, b) \quad (1)$$

$$f(x, x) \approx f(c, x) \quad (2)$$

$$f(x, x) \approx g(x) \quad (3)$$

Apply the Knuth-Bendix completion procedure to E and transform it into a finite convergent term rewrite system; use a Knuth-Bendix ordering with weight 1 for all function symbols and variables and the precedence $f > g > b > c$. Use a reasonable strategy.

Bring your solution to the tutorial on February 5 and compare it with the solution that is discussed there. If you are still unsure afterwards whether your solution is correct or not, feel free to ask the instructor after the tutorial. Your solution will not be graded.