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**Tutorials for "Automated Reasoning"**
**Exercise sheet 10**

**Exercise 10.1:**
Suppose that the atom ordering $\succ$ compares ground atoms by comparing lexicographically first the predicate symbols ($P > Q > R$), then the size of the first argument, and then the size of the second argument (if present). If at least one of the two atoms to be compared is non-ground, $\succ$ compares only the predicate symbols.

Let $N$ be the following set of clauses:

$$P(f(x), x) \vee R(b, b) \qquad (1)$$

$$\neg P(b, x) \vee \neg P(x, b) \vee Q(x) \qquad (2)$$

$$Q(f(b)) \vee \neg Q(b) \vee R(f(x), b) \qquad (3)$$

$$Q(b) \vee \neg R(f(x), f(x)) \qquad (4)$$

$$\neg Q(x) \vee R(x, x) \qquad (5)$$

(a) Which literals are (strictly) maximal in the clauses of $N$?

(b) Which $Res^{\succ}_{sel}$-inferences are possible if *sel* selects no literals? What are their conclusions?

(c) Define a selection function *sel* such that $N$ is saturated under $Res^{\succ}_{sel}$.

(d) Is there a $Res^{\succ}_{sel}$-inference between the clause

$$P(x, f(x)) \vee R(b, b) \qquad (1')$$

and clause (2) if *sel* selects no literals? Why (not)?

**Exercise 10.2:**
Let $\Sigma = (\Omega, \Pi)$ be a signature with $\Omega = \{b/0, f/1\}$ and $\Pi = \{P/1, Q/1\}$. Suppose that the atom ordering $\succ$ compares ground atoms by comparing lexicographically first the predicate symbols ($P > Q$) and then the size of the argument. Let $N$ be the following set of clauses:

$$\neg Q(y) \vee P(y)$$
$$Q(x) \vee Q(f(x))$$

(1) Sketch how the set $G_\Sigma(N)$ of all ground instances of clauses in $N$ looks like. How is it ordered with respect to the clause ordering $\succ_C$?

(2) Construct the candidate interpretation $I^{\succ}_{G_\Sigma(N)}$ of the set of all ground instances of clauses in $N$.

**Exercise 10.3:**
In Sect. 3.14 of the lecture notes, the inference rules for ground resolution with ordering restrictions (without selection functions) are given by

*(Ground) Ordered Resolution:*

$$\frac{D \vee A \qquad C \vee \neg A}{D \vee C} \quad \text{if } A \succ L \text{ for all } L \text{ in } D \text{ and } \neg A \succeq L \text{ for all } L \text{ in } C.$$

*(Ground) Ordered Factorization:*

$$\frac{C \vee A \vee A}{C \vee A} \quad \text{if } A \succeq L \text{ for all } L \text{ in } C.$$

This calculus is sound and refutationally complete for sets of ground clauses.

Suppose that we replace the ordering restriction for the first inference rule by "if $A \succ L$ for all $L$ in $D$ and $A \succeq L$ for all $L$ in $C$."

(a) Is the calculus with this modification still sound? If yes, give a short explanation; if no, give a counterexample.

(b) Is the calculus with this modification still refutationally complete? If yes, give a short explanation; if no, give a counterexample.

Bring your solution to the tutorial on January 13 and compare it with the solution that is discussed there. If you are still unsure afterwards whether your solution is correct or not, feel free to ask the instructor after the tutorial. Your solution will not be graded.