Assignment 1 *(SAT solver applications)* (6 + 6 = 12 points)

A finite graph is a pair (V, E), where V is a finite non-empty set and $E \subseteq V \times V$. The elements of V are called vertices or nodes; the elements of E are called edges. A graph has a 3-coloring, if there exists a function $\phi : V \to \{0, 1, 2\}$ such that for every edge $(v, v') \in E$ we have $\phi(v) \neq \phi(v')$.

Part (a)

Give a linear time translation from finite graphs (V, E) to propositional clause sets N such that (V, E) has a 3-coloring if and only if N is satisfiable and such that every model of N corresponds to a 3-coloring ϕ and vice versa.

Part (b)

A 3-coloring is called complete, if for every pair $(c, c') \in \{0, 1, 2\} \times \{0, 1, 2\}$ with $c \neq c'$ there exists an edge $(v, v') \in E$ such that $\phi(v) = c$ and $\phi(v') = c'$ or $\phi(v) = c'$ and $\phi(v') = c$. Give a linear time translation from finite graphs (V, E) to propositional clause sets N such that (V, E) has a complete 3-coloring if and only if N is satisfiable and such that every model of N corresponds to a complete 3-coloring ϕ and vice versa.

Assignment 2 *(CDCL)* (7 + 7 = 14 points)

Let N be some set of propositional clauses over $\Pi = \{P, Q, R, S, T, U, V, W\}$ that contains, among others, the clauses

$P \vee W$	(1)	$\neg S \vee T \vee \neg W$	(6)
$Q \vee V$	(2)	$\neg R \vee \neg S$	(7)
$R \vee U \vee \neg V$	(3)	$\neg P \vee \neg Q$	(8)
$P \vee \neg T \vee \neg U$	(4)	$S \vee \neg U \vee V$	(9)
$S \vee \neg U \vee \neg W$	(5)		

Suppose that we use the relation $\Rightarrow_{\text{CDCL}}$ to test whether N is satisfiable or not, and that, during the CDCL-derivation, we reach the state

$$\neg P^{\mathrm{d}} \;\; \underset{(1)}{W} \;\; \neg Q^{\mathrm{d}} \;\; \underset{(2)}{V} \;\; \neg R^{\mathrm{d}} \;\; \underset{(3)}{U} \;\; \underset{(4)}{\neg T} \;\; \underset{(5)}{S} \;\; \| \; N$$

where the numbers below the deduced literals indicate the clauses used in the *Unit Propagate* or *Backjump* rule. At this point, clause (6) is a conflict clause.

Part (a)

Compute a suitable backjump clause using the 1UIP method and determine the best possible successor state for that backjump clause.

Part (b)

One of the inprocessing techniques from Sect. 2.9 can be used to show that the clause sets N and $N \setminus \{(9)\}$ are equisatisfiable. Explain which technique can be used and how it works in this case.

Assignment 3 *(Propositional logic)* (12 points)

Let F be a propositional formula and let C be a propositional clause. Prove: If every propositional variable that occurs in F occurs also in C, and if there exists a valuation $\mathcal{A}$ such that both F and C are false under $\mathcal{A}$, then $F \models C$.

Assignment 4 *(Algebras)* (10 + 6 = 16 points)

Let $\Sigma = (\Omega, \Pi)$ be a signature where Π contains two predicate symbols Q and R with the same arity n and possibly further predicate symbols. For any Σ-formula F let $\mathrm{rep}(F)$ be the formula that one obtains by replacing every atom $Q(s_1, \dots, s_n)$ in F by the corresponding atom $R(s_1, \dots, s_n)$.

Part (a)

Prove: If F is valid, then $\mathrm{rep}(F)$ is valid. (It is sufficient if you consider non-equational atoms, disjunctions $G \vee G'$ and negations $\neg G$; the other cases are handled analogously.)

Part (b)

Refute: If F is satisfiable, then $\mathrm{rep}(F)$ is satisfiable.

Assignment 5 *(CNF)* (14 points)

Let $\Sigma = (\{c/0, f/1\}, \{P/4,\ Q/2,\ R/3\})$. Transform the Σ-formula

$$F \;=\; \exists w\, \forall x\, \exists z\, \neg \exists y\, \forall v \Big(\neg P(c, v, f(x), y) \wedge \big(Q(v, z) \rightarrow R(x, z, w) \big) \Big)$$

into clause normal form using the improved algorithm from Section 3.6. (There are no subformulas in F for which one should introduce a definition.)

Assignment 6 *(Clause orderings)* (12 points)

Determine all strict total orderings $\succ$ on the atomic formulas P, Q, R, S such that the associated clause ordering $\succ_{\mathrm{C}}$ satisfies the properties (1)–(3) simultaneously:

$$P \vee Q \succ_{\mathrm{C}} \neg Q \qquad (1)$$

$$R \vee Q \succ_{\mathrm{C}} \neg P \vee \neg P \qquad (2)$$

$$\neg R \vee \neg R \succ_{\mathrm{C}} S \qquad (3)$$