Assignment 1 *(Multisets)* (10 points)

Determine all strict total orderings $\succ$ on the set $\{a, b, c, d, e\}$ such that

$$\{a, b\} \succ_{\text{mul}} \{a, a, c\} \qquad (1)$$
$$\{c, d\} \succ_{\text{mul}} \{b, b, b\} \qquad (2)$$
$$\{a, e\} \succ_{\text{mul}} \{c, e, e\} \qquad (3)$$

Assignment 2 *(Propositional logic)* (12 points)

Prove or refute: If $N = \{C_1, \ldots, C_n\}$ is a finite set of propositional clauses without duplicated literals or complementary literals, and if for every $i \in \{1, \ldots, n\}$ the clause C_i has exactly i literals, then N is satisfiable.

Assignment 3 *(CDCL)* (8 + 4 = 12 points)

Let N be some set of propositional clauses over $\Pi = \{P, Q, R, S, T, U, V, W\}$ that includes the clauses

$\neg P \lor \neg W$	(1)	$\neg R \lor U \lor \neg V$	(4)
$\neg R \lor V \lor W$	(2)	$\neg S \lor \neg U \lor W$	(5)
$\neg Q \lor \neg R \lor \neg T$	(3)	$S \lor \neg V$	(6)

Suppose that we use the relation $\Rightarrow_{\text{CDCL}}$ to test whether N is satisfiable or not, and that, during the CDCL-derivation, we reach the state

$$P^{\text{d}} \ \underset{(1)}{\neg W} \ Q^{\text{d}} \ R^{\text{d}} \ \underset{(2)}{V} \ \underset{(3)}{\neg T} \ \underset{(4)}{U} \ \underset{(5)}{\neg S} \ \| \ N$$

where the numbers below the deduced literals indicate the clauses used in the *Unit Propagate* or *Backjump* rule. At this point, clause (6) is a conflict clause.

Part (a)

Compute a suitable backjump clause using the 1UIP method and determine the best possible successor state for that backjump clause.

Part (b)

Give two *other* backjump clauses that one might use in this situation and determine the best possible successor state for each of these two backjump clauses.

Assignment 4 *(Well-founded orderings, Algebras)* (8 + 4 + 8 = 20 points)

Let $\succ$ be a well-founded strict partial ordering on a set M. A function $\phi : M^n \to M$ with $n \geq 1$ is called strictly monotonic in the j-th argument if $a_j \succ a'_j$ implies $\phi(a_1, \dots, a_j, \dots, a_n) \succ \phi(a_1, \dots, a'_j, \dots, a_n)$ for all arguments $a_1, \dots, a_n, a'_j \in M$.

Part (a) Prove: If the ordering $\succ$ on the set M is well-founded and total, and if $\phi : M^n \to M$ with $n \geq 1$ is strictly monotonic in the j-th argument, then $\phi(a_1, \dots, a_j, \dots, a_n) \succeq a_j$ for all $a_1, \dots, a_n \in M$.

Part (b) In part (a), it was required that $\succ$ is a *total* ordering. Give an example that shows that the property of part (a) does not hold if the ordering $\succ$ is well-founded but not total.

Part (c) Use part (a) to prove the following property: Let $\Sigma = (\Omega, \Pi)$ be a signature, let $\mathcal{A}$ be a Σ-algebra. Let $\succ$ be a well-founded total ordering on the universe $U_\mathcal{A}$ of $\mathcal{A}$, such that $f_\mathcal{A} : U_\mathcal{A}^n \to U_\mathcal{A}$ is strictly monotonic in every argument for every $f/n \in \Omega$ with $n \geq 1$. Let β be an arbitrary $\mathcal{A}$-assignment, let $t \in \mathrm{T}_\Sigma(X)$. Then $\mathcal{A}(\beta)(t) \succeq \beta(x)$ for every variable $x \in \mathrm{var}(t)$.

Assignment 5 *(CNF, Resolution)* (12 points)

Let $\Sigma = (\Omega, \Pi)$ with $\Omega = \{b/0,\ c/0\}$ and $\Pi = \{P/1,\ Q/0,\ R/0\}$. Use the ground resolution calculus *Res* to check whether

$$\{P(b) \to Q,\ P(b) \to R,\ P(c) \to Q\} \models (Q \leftrightarrow R) \lor (P(b) \to P(c))$$

Use the improved CNF transformation of Sect. 3.6 for preprocessing. (There are no subformulas for which one should introduce a definition.)

Assignment 6 *(Clause orderings)* (7 + 7 = 14 points)

Let $\Sigma = (\Omega, \Pi)$ be a signature with $\Omega = \{b/0, f/1\}$ and $\Pi = \{P/1,\ Q/1\}$. Suppose that the atom ordering $\succ$ compares ground atoms by comparing lexicographically first the size of the argument and then the predicate symbols ($P > Q$). Let N be the following set of clauses:

$$\forall x\ (\neg P(x)\ \lor\ P(f(x))) \qquad (1)$$
$$\neg Q(f(b))\ \lor\ P(f^3(b)) \qquad (2)$$
$$Q(b)\ \lor\ Q(f(b)) \qquad (3)$$

where $f^0(b)$ is defined as b and $f^{i+1}(b)$ is defined as $f(f^i(b))$.

Part (a) Sketch how the set $G_\Sigma(N)$ of all ground instances of clauses in N looks like. How is it ordered with respect to the clause ordering $\succ_C$?

Part (b) Construct the candidate interpretation $I^{\succ}_{G_\Sigma(N)}$ of the set of all ground instances of clauses in N. Is it a model of $G_\Sigma(N)$?