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**Tutorials for "Automated Reasoning"**
**Exercise sheet 9**

**Exercise 9.1:**
Let $N$ be the following set of ground clauses:

$$\neg P_3 \vee P_1 \vee P_1 \quad (1)$$
$$\neg P_2 \vee P_1 \quad (2)$$
$$P_4 \vee P_4 \quad (3)$$
$$P_3 \vee \neg P_2 \quad (4)$$
$$P_4 \vee P_3 \quad (5)$$

(a) Find a total atom ordering $\succ$ such that both clause (2) and (5) are redundant w.r.t. $N$.

(b) Prove that there is no atom ordering such that clause (4) is redundant w.r.t. $N$.

**Exercise 9.2:**
Prove that it is undecidable whether a clause $C$ is redundant w. r. t. a set of clauses $N$. (You may use the fact that the satisfiability of a set of first-order clauses is undecidable.)

**Exercise 9.3:**
Prove the details of non-ground case of Thm. 3.45 (ii): If $M \subseteq \mathit{Red}(N)$, then $\mathit{Red}(N) \subseteq \mathit{Red}(N \setminus M)$.

Note: $\mathit{Red}(N)$ denotes the set of all clauses that are redundant w. r. t. a set $N$ of clauses. (This definition is missing in the lecture notes.)

**Exercise 9.4:**
Prove Lemma 3.46: Let $N_0 \vdash N_1 \vdash N_2 \vdash \dots$ be a run. Then $\mathit{Red}(N_i) \subseteq \mathit{Red}(\bigcup_{i \geq 0} N_i)$ and $\mathit{Red}(N_i) \subseteq \mathit{Red}(N_\infty)$ for every $i$.

**Exercise 9.5:**
Let $\Sigma = (\{f/2, b/0, c/0, d/0\}, \emptyset)$; let $E = \{\, \forall x\, (f(x,x) \approx b),\ c \approx d \,\}$; let $X = \{x, y, z\}$ be a set of variables. For any $t \in \mathrm{T}_\Sigma(X)$ let $[t]$ denote the congruence class of $t$ w.r.t. $E$. Let $\mathcal{T} = \mathrm{T}_\Sigma(X)/E$ and let $\beta : X \to \mathcal{T}$ be the assignment that maps every variable to $[c]$.

State for each of the following "items" whether it is (a) a term, (b) a set of terms, (c) a set of ground terms, (d) a congruence class w.r.t. $E$, (e) a formula, (f) a truth value, or (g) non-sensical. (More than one answer may be correct.)

(1) $[c]$
(2) $[b]$
(3) $\{d\}$
(4) $f(c,d)$
(5) $[f(c,d)]$
(6) $b \approx c$
(7) $f([c],[d])$
(8) $f_{\mathcal{T}}(\{c,d\},\{c,d\})$
(9) $\mathcal{T}(x)$
(10) $\mathcal{T}(\beta)(f(c,y))$
(11) $f_{\mathcal{T}}(\mathcal{T}(\beta)(c), \mathcal{T}(\beta)(y))$
(12) $\mathcal{T}(\beta)(\forall x, y\ (f(x,x) \approx f(y,y)))$

Bring your solution to the tutorial on January 24 and compare it with the solution that is discussed there. If you are still unsure afterwards whether your solution is correct or not, feel free to ask the instructor after the tutorial. Your solution will not be graded.