max planck institut
informatik

Universität
des
Saarlandes
FR Informatik

Uwe Waldmann **January 3, 2024**

Tutorials for "Automated Reasoning"
Exercise sheet 7

Exercise 7.1:
Prove the following statement: If N is a set of propositional formulas and C is a propositional formula such that $N \models C$, then there exists a finite subset $M \subseteq N$ such that $M \models C$.

Exercise 7.2:
Using the (i) standard and the (ii) polynomial unification rules, compute most general unifiers of $P(g(x_1, x_1), g(x_3, h(x_4)))$ and $P(g(h(x_2), h(h(x_6))), g(h(x_5), x_5))$, if they exist.

Exercise 7.3:
Using the (i) standard and the (ii) polynomial unification rules, compute most general unifiers of $P(g(x_1, g(f(x_3), x_3)), g(h(x_4), x_3))$ and $P(g(x_2, x_2), g(x_3, h(x_1)))$, if they exist.

Exercise 7.4:
Prove that the relation $\Rightarrow_{PU}$ (rule-based polynomial unification) is terminating. Hint: The first component of the lexicographic combination of orderings used to prove termination of $\Rightarrow_{SU}$ can be kept, but the second one cannot, due to the last rule for $\Rightarrow_{PU}$.

Exercise 7.5:
Challenge Problem: Prove part (ii) of Prop. 3.23: If $\sigma \leq \tau$ and $\tau \leq \sigma$, then there exist variable renamings δ and δ' (i.e., *bijective* substitutions mapping variables to variables), so that $x\sigma\delta = x\tau$ and $x\tau\delta' = x\sigma$ for every x in X. (Note: $\{x \mapsto y\}$ is *not* a bijective substitution!)

Bring your solution to the tutorial on January 10 and compare it with the solution that is discussed there. If you are still unsure afterwards whether your solution is correct or not, feel free to ask the instructor after the tutorial. Your solution will not be graded.