



Uwe Waldmann

December 20, 2023

Tutorials for “Automated Reasoning”
Exercise sheet 6

Exercise 6.1:

Let $\Sigma = (\Omega, \Pi)$ with $\Omega = \{b/0, c/0, d/0\}$ and $\Pi = \{P/1, Q/0\}$. Are the following statements correct? Give a brief explanation.

- (1) The formula $\forall x P(x)$ has infinitely many Σ -models.
- (2) The formula $P(b) \wedge P(c) \wedge \neg P(d)$ has a Σ -model with a two-element universe.
- (3) Every Σ -model of $\neg P(b) \wedge \neg P(c)$ has a finite universe.
- (4) Every Σ -model of $P(b) \wedge P(c) \wedge P(d)$ is a model of $\forall x P(x)$.
- (5) Every Herbrand model over Σ of $P(b) \wedge P(c) \wedge P(d)$ is a model of $\forall x P(x)$.
- (6) The formula $Q \wedge P(c) \wedge \neg P(d)$ has exactly two Herbrand models over Σ .

Exercise 6.2:

Let N be the following set of ground clauses:

$$\{P \vee Q, P \vee \neg Q, \neg P \vee Q, \neg P \vee \neg Q\}$$

- (i) Show that $N \vdash_{Res} \perp$, that is, derive \perp from N using the Resolution and the Positive Factorization rule.
- (ii) Why is it impossible to derive the empty clause from these clauses without using factorization?

Exercise 6.3:

Find a finite set N of ground clauses such that no clause in N is a tautology and such that $Res^*(N)$ is infinite.

Exercise 6.4:

Let N be the set containing the following ground clauses:

$$C_1 = P(a) \vee \neg Q(a) \vee \neg Q(b)$$

$$C_2 = P(a) \vee P(a) \vee Q(b)$$

$$C_3 = P(b) \vee Q(a) \vee Q(b)$$

$$C_4 = Q(a) \vee Q(b) \vee Q(b)$$

$$C_5 = \neg P(a) \vee Q(b)$$

$$C_6 = \neg P(b)$$

Let the ordering on ground atoms be given by $P(a) \succ P(b) \succ Q(a) \succ Q(b)$.

(i) Order the clauses in N according to the associated clause ordering \succ_C .

(ii) Compute the candidate interpretation I_N^\succ . Which clauses are productive, what do they produce, which clause is the minimal counterexample (if it exists)?

Bring your solution to the tutorial on January 3 and compare it with the solution that is discussed there. If you are still unsure afterwards whether your solution is correct or not, feel free to ask the instructor after the tutorial. Your solution will not be graded.