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Tutorials for "Automated Reasoning"
Exercise sheet 5

Exercise 5.1:
Let the signature $\Sigma = (\Omega, \Pi)$ be given by $\Omega = \{+/2, s/1, 0/0\}$ and $\Pi = \emptyset$, and let

$$\begin{aligned} F_1 &= \forall x\,(x + 0 \approx x) \\ F_2 &= \forall x \forall y\,(x + s(y) \approx s(x + y)) \\ F_3 &= \forall x \forall y\,(x + y \approx y + x) \\ F_4 &= \neg \forall x \forall y\,(x + y \approx y + x). \end{aligned}$$

(1) Determine a Σ-algebra $\mathcal{A}$ with an universe of exactly two elements such that $\mathcal{A}$ is a model of F_1, F_2, F_3.

(2) Determine a Σ-algebra $\mathcal{A}$ with an universe of exactly two elements such that $\mathcal{A}$ is a model of F_1, F_2, F_4.

Exercise 5.2:
Prove Prop. 3.5: For any Σ-formula F, $\mathcal{A}(\beta)(F\sigma) = \mathcal{A}(\beta \circ \sigma)(F)$.
(It is sufficient if you prove the property for atomic formulas $P(s_1, \dots, s_n)$, disjunctions $F \vee G$, and universally quantified formulas $\forall x\, F$; the other cases are proved similarly.)

Exercise 5.3:
Transform the first-order formula

$$F \;=\; \forall x\, \exists y\, \exists z \Big(\big(P(x) \wedge R(x, y)\big) \vee \neg \forall w\, Q(z, w) \Big)$$

into clause normal form using the improved algorithm from Section 3.6. (There are no subformulas in F for which one should introduce a definition.)

Bring your solution to the tutorial on Dezember 20 and compare it with the solution that is discussed there. If you are still unsure afterwards whether your solution is correct or not, feel free to ask the instructor after the tutorial. Your solution will not be graded.