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Tutorials for "Automated Reasoning"
Exercise sheet 4

Exercise 4.1:
For any propositional formula F let $negvar(F)$ be the formula obtained from F by replacing every propositional variable by its negation. (E. g., $negvar(P \vee (\neg Q \rightarrow (\neg P \wedge \top))) = \neg P \vee (\neg\neg Q \rightarrow (\neg\neg P \wedge \top))$.) Prove or refute: If a formula F is satisfiable, then $negvar(F)$ is satisfiable. (It is suficient if you consider the boolean connectives $\neg$ and $\wedge$; the others are handled analogously.)

Exercise 4.2:
Let N be the following set of propositional clauses:

$$
\begin{array}{rccccccccccccc}
(1) & \neg P & \vee & Q & \vee & \neg R & & & & & & & & \\
(2) & \neg P & & & & & & & \vee & T & \vee & U & \vee & \neg V \\
(3) & \neg P & \vee & Q & & & & & \vee & \neg T & \vee & \neg U & \vee & V \\
(4) & P & \vee & \neg Q & & & & & & & & & & \\
(5) & & & & & \neg R & & & \vee & \neg T & & & & \\
(6) & & & & & \neg R & & & & & \vee & U & & \\
(7) & P & & & & & \vee & \neg S & & & \vee & U & \vee & V \\
(8) & & & & & R & \vee & \neg S & & & & & & \\
(9) & & & & & R & & & \vee & \neg T & & & \vee & \neg V \\
(10) & & & & & & & S & \vee & \neg T & \vee & \neg U & \vee & V \\
(11) & & & & & & & & & T & \vee & \neg U & & \\
(12) & & & & & & & S & \vee & T & \vee & U & \vee & \neg V \\
(13) & & & & & & & & & & & U & \vee & V
\end{array}
$$

Use the CDCL procedure to check whether N is satisfiable or not; if it is satisfiable, give a model. Use a reasonable strategy. If you use the *Decide* rule, use the largest undefined negative literal according to the ordering $\neg P > \neg Q > \neg R > \neg S > \neg T > \neg U > \neg V$. If you use the *Backjump* rule, determine a suitable backjump clause using the 1UIP method and use the best possible successor state for that backjump clause.

Exercise 4.3:
Prove that the "RAT elimination" rule explained in the *Inprocessing* section is satisfiability-preserving: (*Note: RAT will be explained in the lecture on Dec. 1.*)

C is called an *asymmetric tautology* w. r. t. N, if its negation can be refuted by unit propagation using clauses in N.

We say that C has the *RAT property* w. r. t. N, if it is an asymmetric tautology w. r. t. N, or if there is a literal L in C such that $C = C' \lor L$ and all clauses $D' \lor C'$ for $D' \lor \overline{L} \in N$ are asymmetric tautologies w. r. t. N.

Assume that C has the RAT property w. r. t. N. Show that $N \cup \{C\}$ is satisfiable if and only if N is satisfiable.

Exercise 4.4:
The sudoku puzzle presented in the first lecture has a unique solution.

	1	2	3	4	5	6	7	8	9
1								1	
2	**4**								
3		2							
4					5		4		7
5			8				3		
6			1		9				
7	3			4			2		
8		5		1					
9				8		6			

If we replace the 4 in column 1, row 2, by some other digit, this need no longer hold.

(1) Use a SAT solver to find out for which values in column 1, row 2, the puzzle has no solution.

(2) Describe a set of propositional clauses that is satisfiable if and only if a sudoku puzzle has more than one solution. Use it to find out for which values in column 1, row 2, the puzzle has more than one solution.

Hint: The perl script at `https://rg1-teaching.mpi-inf.mpg.de/autrea-ws23/gensud` produces an encoding of the sudoku above in DIMACS CNF format, which is accepted by most SAT solvers.

Bring your solution to the tutorial on Dezember 6 and compare it with the solution that is discussed there. If you are still unsure afterwards whether your solution is correct or not, feel free to ask the instructor after the tutorial. Your solution will not be graded.