max planck institut
informatik

Universität
des
Saarlandes
FR Informatik

Uwe Waldmann **November 15, 2023**

Tutorials for "Automated Reasoning"
Exercise sheet 3

Exercise 3.1:
Let F be the formula $(Q \rightarrow P) \rightarrow (\neg P \wedge Q \wedge R)$.

(1) Convert F into an equivalent CNF formula as described in Prop. 2.12.

(2) Replace the subformulas $Q \rightarrow P$ and $\neg P \wedge Q \wedge R$ by new variables, add the polarity-dependent definitions for the new variables, and convert again into a CNF formula.

Exercise 3.2:
Prove Proposition 2.14: Let $\mathcal{A}$ be a valuation, let F and G be formulas, and let $H = H[F]_p$ be a formula in which F occurs as a subformula at position p.

If $\mathrm{pol}(H,p) = 1$ and $\mathcal{A}(F) \leq \mathcal{A}(G)$, then $\mathcal{A}(H[F]_p) \leq \mathcal{A}(H[G]_p)$.

If $\mathrm{pol}(H,p) = -1$ and $\mathcal{A}(F) \geq \mathcal{A}(G)$, then $\mathcal{A}(H[F]_p) \leq \mathcal{A}(H[G]_p)$.

(It is sufficient if you consider the boolean connectives $\wedge$ and $\neg$; the other cases are proved analogously. Hint: You must prove both properties simultaneously; it is not possible to prove one of them individually.)

Exercise 3.3:
Suppose that we extend the syntax of propositional formulas by a ternary if-then-else connective and that we define $\mathcal{A}(\text{if } F \text{ then } G_1 \text{ else } G_0)$ as $\mathcal{A}(G_1)$ if $\mathcal{A}(F) = 1$ and as $\mathcal{A}(G_0)$ if $\mathcal{A}(F) = 0$.

(1) How should one extend the definitions of positions and polarities to formulas that include if-then-else? Give an explanation.

(2) There exist several ways to eliminate the if-then-else connective from a formula $H[\text{if } F \text{ then } G_1 \text{ else } G_0]_p$ in a CNF transformation. Which one should be used if p has polarity 1? Which one should be used if p has polarity -1? Give an explanation.

Exercise 3.4:
A partial Π-valuation $\mathcal{A}$ under which all clauses of a clause set N are true is called a partial Π-model of N.

Do the following clause sets over $\Pi = \{P, Q, R\}$ have partial Π-models that are not total Π-models (that is, models in the sense of Sect. 2.3)? If yes, give such a partial Π-model.

(1)
$$\begin{array}{ccccc} P & & & & \\ \neg P & \vee & Q & & \\ \neg P & \vee & \neg Q & \vee & \neg R \end{array}$$

(2)
$$\begin{array}{ccccc} P & & & & \\ \neg P & \vee & Q & & \\ & & \neg Q & \vee & R \\ \neg P & \vee & \neg Q & \vee & \neg R \end{array}$$

(3)
$$\begin{array}{ccccc} P & & & \vee & R \\ \neg P & \vee & Q & \vee & \neg R \\ & & \neg Q & \vee & \neg R \end{array}$$

(4)
$$\begin{array}{ccccc} \neg P & \vee & Q & & \\ & & \neg Q & \vee & R \\ P & & & \vee & \neg R \end{array}$$

Bring your solution to the tutorial on November 29 and compare it with the solution that is discussed there. If you are still unsure afterwards whether your solution is correct or not, feel free to ask the instructor after the tutorial. Your solution will not be graded.

There are no lectures/tutorials on November 22 and 24!