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Tutorials for "Automated Reasoning"
Exercise sheet 2

Exercise 2.1:
Let M be the set $\{a, b, c\}$ and let the ordering $\succ$ be defined by $a \succ b$ and $a \succ c$. (b and c are incomparable!) Consider the following multisets over multisets over M:

(1) $\{\{a, a\}\}$

(2) $\{\{a, b, c\}, \{b, c\}\}$

(3) $\{\{a, b, c\}, \{b, b\}, \{c, c\}\}$

(4) $\{\{a, c\}, \{b, b, c, c\}\}$

Determine for each pair of multisets whether they are comparable with respect to $(\succ_{\text{mul}})_{\text{mul}}$, and, if so, which multiset is larger.

Exercise 2.2:
Prove that the multiset extension $\succ_{\text{mul}}$ of a total ordering $\succ$ is total.

Exercise 2.3:
Which of the following propositional formulas are valid? Which are satisfiable? Which are unsatisfiable?

(1) $\neg P$

(2) $P \rightarrow \bot$

(3) $\bot \rightarrow P$

(4) $(P \vee Q) \rightarrow P$

(5) $P \rightarrow (Q \rightarrow P)$

(6) $Q \rightarrow \neg Q$

(7) $Q \wedge \neg Q$

(8) $\neg(\neg P \wedge \neg\neg P)$

Exercise 2.4:
Let $\Pi = \{P, Q, R\}$. How many models does the Π-formula $(P \wedge Q) \vee (P \wedge \neg R)$ have?

Exercise 2.5:
Let F, G, and H be propositional formulas. Prove or refute:

(1) If $F \models G$ and $G \models H$, then $F \models H$.

(2) If F is satisfiable and G is satisfiable, then $F \wedge G$ is satisfiable.

(3) If F is satisfiable and $F \rightarrow G$ is satisfiable, then G is satisfiable.

(4) If $F \vee G$ is valid, then F is valid or G is satisfiable.

(5) If $F \vee H[F]_p$ is valid, then $F \vee H[\bot]_p$ is valid.

Bring your solution to the tutorial on November 15 and compare it with the solution that is discussed there. If you are still unsure afterwards whether your solution is correct or not, feel free to ask the instructor after the tutorial. Your solution will not be graded.