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**Tutorials for “Automated Reasoning”
Exercise sheet 11**

Exercise 11.1:

Let $\Sigma = (\{f/1, g/2, h/1, b/0, c/0\}, \emptyset)$ and let

$$t_1 = g(h(x), h(c)),$$

$$t_2 = g(x, x),$$

$$t_3 = g(b, f(x)),$$

$$t_4 = f(g(x, y)),$$

$$t_5 = h(g(x, c)).$$

Determine for each $1 \leq i < j \leq 5$ whether t_i and t_j are uncomparable or comparable (and if so, which term is larger) with respect to

- (a) a lexicographic path ordering with precedence $f > g > h > b > c$,
- (b) a Knuth-Bendix-ordering with precedence $h > f > g > b > c$, where h has weight 0, b has weight 3, and all other symbols have weight 1,
- (c) a polynomial ordering over $\{n \in \mathbb{N} \mid n \geq 1\}$ with $P_f(X_1) = X_1 + 1$, $P_g(X_1, X_2) = 2X_1 + X_2 + 1$, $P_h(X_1) = 3X_1$, $P_b = 1$ and $P_c = 3$.

Exercise 11.2:

- (a) Find a polynomial ordering \succ over $\{n \in \mathbb{N} \mid n \geq 1\}$ with linear polynomials such that $g(x) \succ x$, $h(x) \succ g(x)$, and $f(g(x)) \succ g(h(f(x)))$.
- (b) Find a lexicographic path ordering \succ such that $h(h(x)) \succ f(x)$ and $f(g(h(x), y)) \succ h(g(x, f(y)))$.

Bring your solution to the tutorial on February 7 or 9 and compare it with the solution that is discussed there. If you are still unsure afterwards whether your solution is correct or not, feel free to ask the instructor after the tutorial. Your solution will not be graded.