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Tutorials for "Automated Reasoning"
Exercise sheet 10

Exercise 10.1:
Prop. 4.10 states that, if $\to$ is normalizing and confluent, then $b \leftrightarrow^* c \;\Leftrightarrow\; b{\downarrow} = c{\downarrow}$. Prove it by induction on the length of the derivation without using the Church-Rosser Theorem 4.7.

Exercise 10.2:
Compute all critical pairs for each of the following term rewrite systems:

(a) $\{\, f(g(f(x))) \to x,$
$\quad f(g(x)) \to g(f(x)) \,\}$

(b) $\{\, f(x,x) \to b,$
$\quad f(x,g(x)) \to c \,\}$

(c) $\{\, f(f(x,y),z) \to f(x,f(y,z)),$
$\quad f(x,1) \to x \,\}$

(d) $\{\, f(f(x,y),z) \to f(x,f(y,z)),$
$\quad f(1,x) \to x \,\}$

Which systems are locally confluent?

Exercise 10.3:
Is the rewrite system

$$\{\, f(a) \to f(b),\ f(b) \to f(c),\ f(c) \to f(a),\ f(x) \to x \,\}$$

(i) terminating, (ii) normalizing, (iii) locally confluent, (iv) confluent? Give a brief explanation.

Exercise 10.4:
Challenge Problem: Find a signature Σ containing at least one constant symbol, a set E of Σ-equations, and two terms $s, t \in \mathrm{T}_\Sigma(X)$ such that

$$\mathrm{T}_\Sigma(\{x_1\})/E \models \forall \vec{x}(s \approx t),$$

but

$$\mathrm{T}_\Sigma(\{x_1, x_2\})/E \not\models \forall \vec{x}(s \approx t)$$

where $\vec{x}$ consists of all the variables occurring in s and t. (The variables in $\vec{x}$ need not be contained in $\{x_1, x_2\}$.)

Bring your solution to the tutorial on January 31 and compare it with the solution that is discussed there. If you are still unsure afterwards whether your solution is correct or not, feel free to ask the instructor after the tutorial. Your solution will not be graded.