Assignment 1 *(CDCL)* (8 + 4 + 4 = 16 points)

Let N be the following set of propositional clauses:

$$
\begin{array}{ccccccccccccr}
P & & & & & \vee & S & & & & & & (1)\\
P & \vee & Q & & & & & \vee & \neg T & & & & (2)\\
 & & Q & & & \vee & \neg S & & & \vee & \neg U & & (3)\\
 & & \neg Q & \vee & R & \vee & S & \vee & \neg T & \vee & U & & (4)\\
 & & & & R & \vee & \neg S & & & & & & (5)\\
 & & & & R & & & \vee & T & & & & (6)\\
\neg P & & & & & \vee & S & & & \vee & \neg U & & (7)\\
 & & & & \neg R & \vee & \neg S & & & & & & (8)\\
 & & & & \neg R & & & \vee & T & \vee & U & & (9)\\
\neg P & & & \vee & \neg R & \vee & S & \vee & \neg T & & & & (10)
\end{array}
$$

Part (a)

Use the CDCL procedure to compute a model of N.

Part (b)

Use the final state of the CDCL procedure in part (a) to determine the number of models of N in which P is true. Give a brief explanation.

Part (c)

Use the CDCL procedure to show that N has no model in which P is false.

For both part (a) and (c):
Use the CDCL inference rules with a reasonable strategy. If you use the *Decide* rule, use the largest undefined positive literal according to the ordering $P > Q > R > S > T > U$. If you use the *Backjump* rule, determine a suitable backjump clause using the 1UIP method and use the best possible successor state for that backjump clause.

Assignment 2 *(Resolution)* (8 + 8 = 16 points)

Let $\Sigma = (\Omega, \Pi)$ be a signature, let N be a set of ground Σ-clauses.

Part (a)

Prove or refute: All clauses in $Res^*(N)$ are tautologies if and only if all clauses in N are tautologies.

Part (b)

Prove or refute: $Res(N)$ is satisfiable if and only if N is satisfiable.

Assignment 3 *(Propositional Logic)* (12 points)

Let Π be a set of propositional variables. Let N and N' be sets of clauses over Π. Let S be a set of literals that does not contain any complementary literals. Prove: If every clause in N contains at least one literal L with $L \in S$ and if no clause in N' contains a literal L with $\overline{L} \in S$, then $N \cup N'$ is satisfiable if and only if N' is satisfiable.

Assignment 4 *(First-order Logic, Semantics)* (12 points)

Let $\Sigma = (\Omega, \Pi)$ be a first-order signature with $\Omega = \{b/0,\ f/1\}$ and $\Pi = \{P/1\}$. Decide for each of the following statements whether they are true or false:

(1) There is a Σ-model $\mathcal{A}$ of $P(b) \land \neg P(f(b))$ such that $U_{\mathcal{A}} = \{7, 8, 9\}$.

(2) There is a Σ-model $\mathcal{A}$ of $P(b) \land \neg P(f(f(b)))$ such that $f_{\mathcal{A}}(a) = a$ for every $a \in U_{\mathcal{A}}$.

(3) $P(b) \land \neg P(f(b))$ has a Herbrand model.

(4) $P(b) \land \forall x\, \neg P(x)$ has a Herbrand model.

(5) $\forall x\, P(f(x))$ has a Herbrand model with a two-element universe.

(6) $\forall x\, P(x)$ has exactly one Herbrand model.

(7) $\forall x\, P(f(x))$ entails $\forall x\, P(f(f(x)))$.

(Note on grading: A yes/no answer is sufficient; you do not have to give an explanation. However, you need at least four correct answers to get any points for this assignment. Missing answers count like false answers.)

Assignment 5 *(First-order Logic, CNF Transformation)* (12 points)

Let $\Sigma = (\{c/0, f/1\}, \{P/4,\ Q/4\})$. Transform the Σ-formula

$$F \;=\; \exists v\, \forall x\, \forall y\, \forall z\, \neg \forall w \Big(\neg P(c, w, z, x) \land Q(w, y, f(x), v) \Big)$$

into clause normal form using the improved algorithm from Section 3.6. (There are no subformulas in F for which one should introduce a definition.)

Assignment 6 *(Clause Orderings)* (12 points)

Find a strict total ordering $\succ$ on the ground atoms $P(b)$, $P(c)$, Q, R such that

$$P(b) \lor \neg P(c) \;\succ_C\; \neg P(b) \lor P(c) \qquad (1)$$

$$P(b) \lor P(b) \lor P(b) \lor R \;\succ_C\; P(b) \lor R \lor R \qquad (2)$$

$$\neg P(b) \lor Q \;\succ_C\; P(c) \lor R \qquad (3)$$