Assignment 1 *(CDCL)* (14 points)

A Horn clause is a clause with at most one positive literal. Let N be a finite set of propositional Horn clauses and let $\varepsilon \parallel N \Rightarrow^*_{\text{CDCL}} M \parallel N$ be a CDCL derivation in which only the "Unit Propagate" rule is used. Prove: If neither "Unit Propagate" nor "Fail" can be applied to $M \parallel N$, then N is satisfiable.

Assignment 2 *(Ordered resolution)* (8 + 4 + 4 = 16 points)

Let $\Sigma = (\{f/1,\ g/2,\ b/0,\ c/0\},\ \{P/2,\ Q/1,\ R/2\})$; let the atom ordering $\succ$ be a Knuth-Bendix ordering with weight 1 for all function and predicate symbols and variables and the precedence $P > Q > R > f > g > b > c$; let N be the following set of clauses over Σ:

$$\neg R(b, x) \qquad (1)$$

$$Q(g(x, z)) \ \vee\ Q(g(b, f(y))) \ \vee\ \neg R(x, z) \qquad (2)$$

$$P(f(x), x) \ \vee\ \neg Q(f(c)) \ \vee\ \neg R(x, c) \qquad (3)$$

$$\neg P(x, c) \ \vee\ Q(x) \qquad (4)$$

$$\neg P(x, b) \ \vee\ R(c, y) \qquad (5)$$

Part (a) Suppose that the selection function *sel* selects no literals. Compute all $\text{Res}^{\succ}_{sel}$ inferences between the clauses (1)–(5). (Do not compute inferences between derived clauses. Do not compute any inferences that violate the restrictions of the calculus.)

Part (b) Is the set N saturated up to redundancy? Give a brief explanation.

Part (c) Does a selection function *sel* exist for which there are no $\text{Res}^{\succ}_{sel}$ inferences between the clauses (1)–(5)? If yes, which literals does *sel* select in the clauses (1)–(5)?

Assignment 3 *(E-Algebras)* (4 + 6 = 10 points)

Let $\Sigma = (\Omega, \emptyset)$ with $\Omega = \{f/1,\ b/0,\ c/0,\ d/0\}$; let E be the set of (implicitly universally quantified) equations $\{\, f(f(x)) \approx b \,\}$.

Part (a) Show that $b \leftrightarrow^*_E f(b)$. How does the rewrite proof look?

Part (b) Is the universe of the initial E-algebra $\mathrm{T}_\Sigma(\emptyset)/E$ finite or infinite? If it is finite, how many elements does it have?

Assignment 4 *(LPO)* (14 points)

Let $\Sigma = (\Omega, \emptyset)$ be a finite signature, let h be a unary function symbol in Ω, and let $\succ$ be a strict partial ordering ("precedence") on Ω such that h is the smallest element of Ω w.r.t. $\succ$.
Prove: For all terms $s, t \in \mathrm{T}_\Sigma(X)$, we have $s \succ_{\mathrm{lpo}} t$ if and only if $s \succeq_{\mathrm{lpo}} h(t)$.

Assignment 5 *(Knuth–Bendix completion)* (14 points)

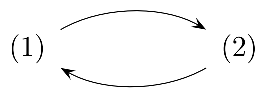
Let E be the following set of equations over $\Sigma = (\{f/1,\ g/1,\ h/1,\ b/0\}, \emptyset)$.

$$f(g(x)) \approx h(x) \qquad (1)$$
$$g(f(x)) \approx h(x) \qquad (2)$$

Apply the Knuth-Bendix completion procedure to E and transform it into a finite convergent term rewrite system; use a Knuth-Bendix ordering with weight 1 for all function symbols and variables and the precedence $f > g > h > b$. Use a reasonable strategy.

Assignment 6 *(Dependency pairs)* (4 + 4 + 4 = 12 points)

Part (a) Find an example of a terminating rewrite system R with exactly two rewrite rules such that $\mathrm{DP}(R)$ contains exactly two dependency pairs and the dependency graph of R has the shape

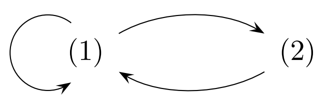


Specify both R and $\mathrm{DP}(R)$.

Part (b) Find an example of a terminating rewrite system R with exactly two rewrite rules such that $\mathrm{DP}(R)$ contains exactly two dependency pairs and the dependency graph of R has the shape

Specify both R and $\mathrm{DP}(R)$.

Part (c) Find an example of a terminating rewrite system R with exactly two rewrite rules such that $\mathrm{DP}(R)$ contains exactly two dependency pairs and the dependency graph of R has the shape



Specify both R and $\mathrm{DP}(R)$.