max planck institut
informatik

**Universität
des
Saarlandes**

FR Informatik

Uwe Waldmann   **January 5, 2022**

**Tutorials for "Automated Reasoning"
Exercise sheet 9**

Exercise 9.1: *(3+3 P)*
Let N be the following set of ground clauses:

$$\neg P_3 \vee P_1 \vee P_1 \qquad (1)$$
$$\neg P_2 \vee P_1 \qquad (2)$$
$$P_4 \vee P_4 \qquad (3)$$
$$P_3 \vee \neg P_2 \qquad (4)$$
$$P_4 \vee P_3 \qquad (5)$$

(a) Find a total atom ordering $\succ$ such that both clause (2) and (5) are redundant w.r.t. N.

(b) Prove that there is no atom ordering such that clause (4) is redundant w.r.t. N.

Exercise 9.2: *(4 P)*
Prove that it is undecidable whether a clause C is redundant w.r.t. a set of clauses N.
(You may use the fact that the satisfiability of a set of first-order clauses is undecidable.)

Exercise 9.3: *(5 P)*
Prove Lemma 3.47: Let $N_0 \vdash N_1 \vdash N_2 \vdash \ldots$ be a run. Then $Red(N_i) \subseteq Red(N_\infty)$ and $Red(N_i) \subseteq Red(N_*)$ for every i.

Note: $Red(N)$ denotes the set of all clauses that are redundant w.r.t. a set N of clauses. (This definition is missing in the lecture notes.)

Exercise 9.4: *(5 P)*
Let N be a set of ground clauses, let $\succ$ be a total and well-founded atom ordering. Prove or refute: If every clause in N is redundant with respect to N, then every clause in N is a tautology.

Send your solution in PDF format via e-mail to your tutor(s) until January 12, 18:00.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.