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Tutorials for "Automated Reasoning"
Exercise sheet 8

Exercise 8.1: (*3 P*)
Use the resolution calculus to show that

$$\Big\{(P \leftrightarrow (Q \land R)),\ (P \leftrightarrow Q)\Big\} \models Q \rightarrow R$$

(You need some preprocessing.)

Exercise 8.2: (*3+3+3+2 P*)
Suppose that the atom ordering $\succ$ compares ground atoms by comparing lexicographically first the predicate symbols ($P > Q > R$), then the size of the first argument, and then the size of the second argument (if present). If at least one of the two atoms to be compared is non-ground, $\succ$ compares only the predicate symbols.

Let N be the following set of clauses:

$$P(f(x), x) \lor R(b, b) \qquad (1)$$
$$\neg P(b, x) \lor \neg P(x, b) \lor Q(x) \qquad (2)$$
$$Q(f(b)) \lor \neg Q(b) \lor R(f(x), b) \qquad (3)$$
$$Q(b) \lor \neg R(f(x), f(x)) \qquad (4)$$
$$\neg Q(x) \lor R(x, x) \qquad (5)$$

(a) Which literals are (strictly) maximal in the clauses of N?

(b) Which $Res^{\succ}_{sel}$-inferences are possible if *sel* selects no literals? What are their conclusions?

(c) Define a selection function *sel* such that N is saturated under $Res^{\succ}_{sel}$.

(d) Is there a $Res^{\succ}_{sel}$-inference between the clause

$$P(x, f(x)) \lor R(b, b) \qquad (1')$$

and clause (2) if *sel* selects no literals? Why (not)?

Exercise 8.3: *(3+3 P)*
Let $\Sigma = (\Omega, \Pi)$ be a signature with $\Omega = \{b/0, f/1\}$ and $\Pi = \{P/1,\ Q/1\}$. Suppose that the atom ordering $\succ$ compares ground atoms by comparing lexicographically first the predicate symbols ($P > Q$) and then the size of the argument. Let N be the following set of clauses:

$$\neg Q(y) \vee P(y)$$

$$Q(x) \vee Q(f(x))$$

(a) Sketch how the set $G_\Sigma(N)$ of all ground instances of clauses in N looks like. How is it ordered with respect to the clause ordering $\succ_C$?

(b) Construct the candidate interpretation $I^{\succ}_{G_\Sigma(N)}$ of the set of all ground instances of clauses in N.

Challenge Problem: *(6 Bonus Points)*
Prove part (ii) of Prop. 3.24: If $\sigma \leq \tau$ and $\tau \leq \sigma$, then there exist variable renamings δ and δ' (i.e., *bijective* substitutions mapping variables to variables), so that $x\sigma\delta = x\tau$ and $x\tau\delta' = x\sigma$ for every x in X. (Note: $\{x \mapsto y\}$ is *not* a bijective substitution, since $x\{x \mapsto y\} = y\{x \mapsto y\}$.)

Submit your solution in lecture hall E1.3, Room 002 during the lecture on January 5 or send it in PDF format via e-mail to your tutor(s) until January 5, 18:00.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.