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Tutorials for "Automated Reasoning"
Exercise sheet 5

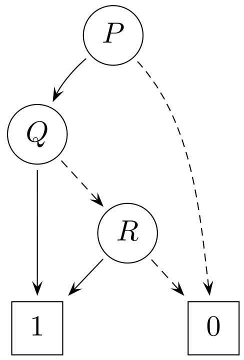
Exercise 5.1: (*5 P*)
For any propositional formula F let $\mathit{negvar}(F)$ be the formula obtained from F by replacing every propositional variable by its negation. (E. g., $\mathit{negvar}(P \vee (\neg Q \rightarrow (\neg P \wedge \top))) = \neg P \vee (\neg\neg Q \rightarrow (\neg\neg P \wedge \top))$.)

Prove or refute: If a formula F is satisfiable, then $\mathit{negvar}(F)$ is satisfiable. (It is sufficient if you consider propositional variables P, negations $\neg F$, and conjunctions $F \wedge G$; the other boolean connectives are handled analogously.)

Exercise 5.2: (*2+1+2 P*)
(1) Give a propositional formula F that is represented by the reduced OBDD on the right.

(2) How many different reduced OBDDs over the propositional variables $\{P, Q, R\}$ have exactly one interior (non-leaf) node?

(3) Find a propositional formula G over the propositional variables $\{P, Q, R\}$, such that the reduced OBDD for G has three interior nodes and the reduced OBDD for $F \vee G$ has one interior node. Give the reduced OBDDs for G and $F \vee G$.



Exercise 5.3: (*2+3 P*)
Let the signature $\Sigma = (\Omega, \Pi)$ be given by $\Omega = \{+/2, s/1, 0/0\}$ and $\Pi = \emptyset$, and let

$$\begin{aligned} F_1 &= \forall x\,(x + 0 \approx x) \\ F_2 &= \forall x \forall y\,(x + s(y) \approx s(x + y)) \\ F_3 &= \forall x \forall y\,(x + y \approx y + x) \\ F_4 &= \neg \forall x \forall y\,(x + y \approx y + x). \end{aligned}$$

(1) Determine a Σ-algebra $\mathcal{A}$ with an universe of exactly two elements such that $\mathcal{A}$ is a model of F_1, F_2, F_3.

(2) Determine a Σ-algebra $\mathcal{A}$ with an universe of exactly two elements such that $\mathcal{A}$ is a model of F_1, F_2, F_4.

Exercise 5.4: (*5 P*)
Prove Prop. 3.5: For any Σ-formula F, $\mathcal{A}(\beta)(F\sigma) = \mathcal{A}(\beta \circ \sigma)(F)$.
(It is sufficient if you prove the property for atomic formulas $P(s_1, \dots, s_n)$, disjunctions $F \vee G$, and universally quantified formulas $\forall x\, F$; the other cases are proved similarly.)

Submit your solution in lecture hall E1.3, Room 002 during the lecture on December 1 or send it in PDF format via e-mail to your tutor(s) until December 1, 18:00.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.