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Tutorials for "Automated Reasoning"
Exercise sheet 4

Exercise 4.1: (*7 P*)
Let N be the following set of propositional clauses:

$$
\begin{array}{lllllllllllllllllll}
(1) & \neg P & \vee & & Q & \vee & \neg R & & & & & & & & & & & & \\
(2) & \neg P & & & & & & & & & \vee & T & \vee & U & \vee & \neg V & & & \\
(3) & \neg P & \vee & & Q & & & & & & \vee & \neg T & \vee & \neg U & \vee & V & & & \\
(4) & P & \vee & \neg Q & & & & & & & & & & & & & & & \\
(5) & & & & & & \neg R & & & & \vee & \neg T & & & & & & & \\
(6) & & & & & & \neg R & & & & & & \vee & U & & & & & \\
(7) & P & & & & & & \vee & \neg S & & & & \vee & U & \vee & V & & & \\
(8) & & & & & R & \vee & \neg S & & & & & & & & & & & \\
(9) & & & & & R & & & & & \vee & \neg T & & & \vee & \neg V & & & \\
(10) & & & & & & & S & \vee & \neg T & \vee & \neg U & \vee & V & & & & & & \\
(11) & & & & & & & & & T & \vee & \neg U & & & & & & & \\
(12) & & & & & & & S & \vee & T & \vee & U & \vee & \neg V & & & & & \\
(13) & & & & & & & & & & & U & \vee & V & & & & &
\end{array}
$$

Use the CDCL procedure to check whether N is satisfiable or not; if it is satisfiable, give a model. Use the CDCL inference rules with a reasonable strategy (i.e., use *Fail* or *Backjump* if possible, otherwise use *Unit Propagate* if possible, otherwise use *Decide*). If you use the *Decide* rule, use the largest undefined negative literal according to the ordering $\neg P > \neg Q > \neg R > \neg S > \neg T > \neg U > \neg V$. If you use the *Backjump* rule, determine a suitable backjump clause using the 1UIP method and use the best possible successor state for that backjump clause.

Exercise 4.2: (*3 P*)
The "Purity deletion" rule explained in Sect. 2.9 is subsumed by other inprocessing rules. By which one(s)? Why?

Exercise 4.3: (*4 P*)
Prove that the "RAT elimination" rule explained in Sect. 2.9 is satisfiability-preserving:

C is called an *asymmetric tautology* w. r. t. N, if its negation can be refuted by unit propagation using clauses in N.

We say that C has the *RAT property* w. r. t. N, if it is an asymmetric tautology w. r. t. N, or if there is a literal L in C such that $C = C' \vee L$ and all clauses $D' \vee C'$ for $D' \vee \overline{L} \in N$ are asymmetric tautologies w. r. t. N.

Assume that C has the RAT property w. r. t. N. Show that $N \cup \{C\}$ is satisfiable if and only if N is satisfiable.

Exercise 4.4: (*2+4 P*)
The sudoku puzzle presented in the first lecture

	1	2	3	4	5	6	7	8	9
1								1	
2	**4**								
3		2							
4					5		4		7
5			8				3		
6			1		9				
7	3			4			2		
8		5		1					
9				8		6			

has a unique solution. If we replace the 4 in column 1, row 2, by some other digit, this need no longer hold.

(1) Use a SAT solver to find out for which values in column 1, row 2, the puzzle has no solution.

(2) Describe a set of propositional clauses that is satisfiable if and only if a sudoku puzzle has more than one solution. Use it to find out for which values in column 1, row 2, the puzzle has more than one solution.

Hint: The perl script at `https://rg1-teaching.mpi-inf.mpg.de/autrea-ws21/gensud` produces an encoding of the sudoku above in DIMACS CNF format, which is accepted by most SAT solvers.

Submit your solution in lecture hall E1.3, Room 002 during the lecture on November 24 or send it in PDF format via e-mail to your tutor(s) until November 24, 18:00.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.