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Tutorials for "Automated Reasoning"
Exercise sheet 2

Exercise 2.1: $(3+4\ P)$
Two ordered sets $(M_1, \succ_1)$ and $(M_2, \succ_2)$ are called order-isomorphic, if there exists a bijection $\varphi : M_1 \to M_2$ such that for all $x, y \in M_1$: $x \succ_1 y$ if and only if $\varphi(x) \succ_2 \varphi(y)$.

Let the ordering $\succ$ over $\{a, b\}$ be defined by $a \succ b$.

(1) The set of finite multisets over $\{a, b\}$ ordered by $\succ_{\mathrm{mul}}$ is not order-isomorphic to $(\mathbb{N}, >)$. Why? Give an explanation. (Hint: Consider the set of all multisets that are smaller than $\{a\}$.)

(2) The set of finite multisets over $\{a, b\}$ ordered by $\succ_{\mathrm{mul}}$ is order-isomorphic to another ordered set that has been discussed in the lecture. Which one? Explain.

Exercise 2.2: $(4\ P)$
Which of the following propositional formulas are valid? Which are satisfiable? Which are unsatisfiable?

(1) $\neg P$

(2) $P \to \bot$

(3) $\bot \to P$

(4) $(P \vee Q) \to P$

(5) $P \to (Q \to P)$

(6) $Q \to \neg Q$

(7) $Q \wedge \neg Q$

(8) $\neg(\neg P \wedge \neg\neg P)$

Exercise 2.3: $(3\ P)$
Let $\Pi = \{P, Q, R\}$. How many models does the Π-formula $(P \wedge Q) \vee (P \wedge \neg R)$ have?

Exercise 2.4: (*6 P*)
Let F, G, and H be propositional formulas. Prove or refute:

(1) If $F \models G$ and $G \models H$, then $F \models H$.

(2) If F is satisfiable and G is satisfiable, then $F \wedge G$ is satisfiable.

(3) If F is satisfiable and $F \rightarrow G$ is satisfiable, then G is satisfiable.

(4) If $F \vee G$ is valid, then F is valid or G is satisfiable.

(5) If $F \vee G[F]_p$ is valid, then $F \vee G[\bot]_p$ is valid.

Submit your solution in lecture hall E1.3, Room 002 during the lecture on November 10 or send it in PDF format via e-mail to your tutor(s) until November 10, 18:00.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.