max planck institut
informatik

Universität
des
Saarlandes
FR Informatik

Uwe Waldmann **February 2, 2022**

Tutorials for "Automated Reasoning"
Exercise sheet 13

Exercise 13.1:
Let $\Sigma = (\Omega, \emptyset)$ with $\Omega = \{f/4,\ b/0,\ c/0,\ d/0,\ e/0\}$. Let $\succ$ be an LPO with the precedence $f > b > c > d > e$. Let E be the set of equations

$$f(w, x, y, z) \approx f(x, y, z, w) \qquad (1)$$
$$f(c, d, e, b) \approx b \qquad (2)$$
$$f(c, b, e, d) \approx c \qquad (3)$$

Compute the set of semi-critical pairs $\mathrm{SC}_\succ(E)$.

Exercise 13.2:
Let $\Sigma = (\Omega, \emptyset)$ with $\Omega = \{b/0,\ f/1,\ g/1\}$. Which ground terms are in T_∞ for the following TRS?

$$f(f(b)) \to g(b) \qquad (1)$$
$$g(x) \to g(f(x)) \qquad (2)$$

Exercise 13.3:
Let $\Sigma = (\Omega, \emptyset)$ with $\Omega = \{f/2,\ g/2,\ h/1,\ k/1,\ b/0\}$.

(a) Compute the dependency pairs of the following rewrite system R over Σ:

$$f(x, h(x)) \to h(k(x)) \qquad (1)$$
$$f(h(x), y) \to g(x, g(h(x), x)) \qquad (2)$$
$$g(x, x) \to f(x, x) \qquad (3)$$
$$g(x, y) \to y \qquad (4)$$
$$h(b) \to b \qquad (5)$$

(b) Compute the approximated dependency graph for R (using cap and ren).

(c) Use the subterm criterion to show that R is terminating. If a graph is modified, depict both the original and the modified graph and indicate the strongly connected components in the graphs.

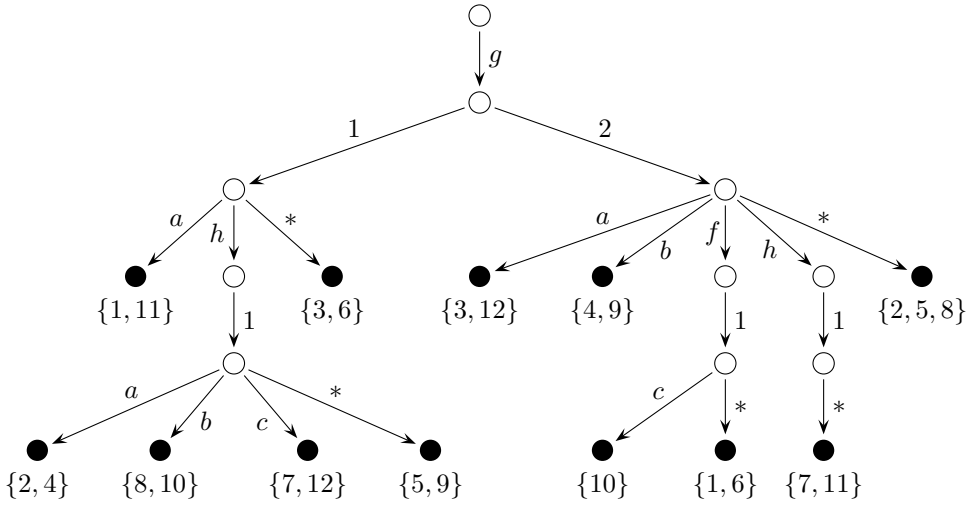
(d) The approximated dependency graph contains an edge from a dependency pair generated by rule (3) to a dependency pair generated by rule (1). Is this edge also contained in the exact dependency graph? Give an explanation.

Exercise 13.4:
The approximated dependency graph that is described in Sect. 5.1 contains an edge from $s \to t$ to $u \to v$ if $\mathrm{ren}(\mathrm{cap}(t))$ and u are unifiable. Suppose that we change this definition so that ren and cap are not only applied to t but also to u. Would this be sound? Would this be useful? Give an explanation.

Exercise 13.5:
[Note: This exercise relies on material that will be discussed in the lecture on February 7.]

(a) Does the path index below contain the terms $t_1 = g(h(c), h(*))$, $t_2 = g(h(a), b)$, $t_3 = g(*, *)$? If yes, what are their numbers in the index?

(b) Find the numbers of all terms in the path index that are generalizations of $s = g(h(a), f(*))$ (that is, terms t such that $s = t\sigma$).



Bring your solution to the lecture/Q&A session on February 9. By lack of time, it will *not* be checked by the tutors.