



Uwe Waldmann

January 26, 2022

**Tutorials for “Automated Reasoning”**  
**Exercise sheet 12**

**Exercise 12.1:** (3 P)

Let  $\Sigma = (\{f/2, g/1, h/1, b/0, c/0\}, \emptyset)$  and let

$$t_1 = h(f(x, g(b))),$$

$$t_2 = g(g(g(g(c)))),$$

$$t_3 = f(h(x), g(b)),$$

$$t_4 = f(g(x), x).$$

Determine for each  $1 \leq i < j \leq 4$  whether  $t_i$  and  $t_j$  are incomparable or comparable (and if so, which term is larger) with respect to a Knuth–Bendix ordering with precedence  $h > f > g > b > c$ , where  $h$  has weight 0 and all other symbols have weight 1,

**Exercise 12.2:** (2+2+3 P)

- Find a polynomial ordering  $\succ$  over  $\{n \in \mathbb{N} \mid n \geq 1\}$  with linear polynomials such that  $g(x) \succ x$ ,  $h(x) \succ g(x)$ , and  $f(g(x)) \succ g(h(f(x)))$ .
- Find a lexicographic path ordering  $\succ$  such that  $h(h(x)) \succ f(x)$  and  $f(g(h(x), y)) \succ h(g(x, f(y)))$ .
- Find a Knuth–Bendix ordering  $\succ$  such that the set of clauses

$$P(f(x, y), y) \vee P(g(y), g(x)) \quad (1)$$

$$\neg P(f(b, y), y) \vee \neg Q(y, y) \quad (2)$$

$$Q(g(x), g(y)) \vee \neg Q(x, h(y)) \quad (3)$$

is saturated under  $Res_{sel}^\succ$ , where  $sel$  does not select any literals.

**Exercise 12.3:** (5 P)

Apply the Knuth–Bendix procedure to the set of equations

$$f(f(x)) \approx g(x) \quad (1)$$

$$f(b) \approx c \quad (2)$$

and transform it into a finite convergent term rewrite system; use the Knuth–Bendix ordering with weight 1 for all function symbols and variables and the precedence  $g > f > b > c$ .

**Exercise 12.4:** (5 P)

Apply the Knuth–Bendix procedure to the set of equations

$$f(0, f(x, f(y, z))) \approx f(f(0, x), f(y, z)) \quad (1)$$

$$f(0, x) \approx 0 \quad (2)$$

$$f(x, 1) \approx x \quad (3)$$

and transform it into a finite convergent term rewrite system; use the Knuth–Bendix ordering with weight 1 for all function symbols and variables and the precedence  $f > 1 > 0$ . Start by orienting the first equation.

**Challenge Problem:** (5 Bonus Points)

Find a signature  $\Sigma$  containing at least one constant symbol, a set  $E$  of  $\Sigma$ -equations, and two terms  $s, t \in T_{\Sigma}(X)$  such that

$$T_{\Sigma}(\{x_1\})/E \models \forall \vec{x}(s \approx t),$$

but

$$T_{\Sigma}(\{x_1, x_2\})/E \not\models \forall \vec{x}(s \approx t)$$

where  $\vec{x}$  consists of all the variables occurring in  $s$  and  $t$ . (The variables in  $\vec{x}$  need not be contained in  $\{x_1, x_2\}$ .)

Send your solution in PDF format via e-mail to your tutor(s) until February 2, 18:00.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.