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Tutorials for "Automated Reasoning"
Exercise sheet 11

Exercise 11.1: (*5 P*)
Compute all critical pairs for each of the following term rewrite systems. (Omit the trivial critical pairs obtained by overlapping a rule with itself at the position ε.)

(a) $\{\, f(g(f(x))) \to x,$
$f(g(x)) \to g(f(x)) \,\}$

(b) $\{\, f(x, x) \to b,$
$f(x, g(x)) \to c \,\}$

(c) $\{\, f(g(x)) \to x,$
$f(c) \to c \,\}$

(d) $\{\, f(f(x, y), z) \to f(x, f(y, z)),$
$f(x, 1) \to x \,\}$

(e) $\{\, f(f(x, y), z) \to f(x, f(y, z)),$
$f(1, x) \to x \,\}$

Which systems are locally confluent?

Exercise 11.2: (*5 P*)
Prove Thm. 4.30: If the precedence $\succ$ is total, then the lexicographic path ordering $\succ_{\text{lpo}}$ is total on ground terms, i.e., for all $s, t \in \mathrm{T}_\Sigma(\emptyset)$: $s \succ_{\text{lpo}} t \;\vee\; t \succ_{\text{lpo}} s \;\vee\; s = t$.

Exercise 11.3: (*3 P*)
Let $\Sigma = (\{f/1,\ g/2,\ h/1,\ b/0,\ c/0\}, \emptyset)$ and let

$$\begin{aligned} t_1 &= g(h(x), c), \\ t_2 &= g(x, h(h(x))), \\ t_3 &= h(g(x, b)), \\ t_4 &= f(g(h(h(x)), y)). \end{aligned}$$

Determine for each $1 \leq i < j \leq 4$ whether t_i and t_j are uncomparable or comparable (and if so, which term is larger) with respect to a polynomial ordering over $\{\, n \in \mathbb{N} \mid n \geq 1 \,\}$ with $P_f(X_1) = X_1 + 1$, $P_g(X_1, X_2) = X_1 + X_2$, $P_h(X_1) = 2X_1$, $P_b = 1$ and $P_c = 3$.

Exercise 11.4: (*3 P*)
Let $\Sigma = (\{f/2,\ g/2,\ h/1,\ b/0,\ c/0\}, \emptyset)$ and let

$$\begin{aligned} t_1 &= h(h(b)), \\ t_2 &= g(c, g(c, c)), \\ t_3 &= f(h(x), y), \\ t_4 &= f(x, b), \end{aligned}$$

Determine for each $1 \leq i < j \leq 4$ whether t_i and t_j are uncomparable or comparable (and if so, which term is larger) with respect to a lexicographic path ordering with the precedence $f \succ b \succ g \succ h \succ c$.

Challenge Problem: (*5 Bonus Points*)
Let $(A, \rightarrow)$ be a reduction system such that for every $a, b, c \in A$, whenever $b \leftarrow a \rightarrow c$ then $b = c$ or there is a $d \in A$ such that $b \rightarrow d \leftarrow c$.

Show that if an element $a \in A$ has a normal form, then there is no infinite reduction sequence starting from a. (Hint: You have to strengthen the statement in order to get a property that can be proved by induction.)

Send your solution in PDF format via e-mail to your tutor(s) until January 26, 18:00.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.