Assignment 1 *(CNF transformation)* (12 points)

Transform the first-order formula

$$F \ = \ \exists z\, \forall x\, \Big(\big(\exists y\, P(z,y)\big) \leftrightarrow \big(\exists y\, Q(x,y)\big) \Big)$$

into clause normal form using the improved algorithm from Section 3.6. (There are no subformulas in F for which one should introduce a definition.)
Are the formula F and its clause normal form equivalent? Give a brief explanation.

Assignment 2 *(Ordered resolution with selection)* (4 + 10 = 14 points)

Let $\Sigma = (\{f/1,\ g/1,\ h/1,\ b/0,\ c/0\},\ \{P/2\})$; let N be the following set of clauses over Σ:

$$P(f(x), f(x)) \qquad (1)$$

$$P(g(x), g(x)) \qquad (2)$$

$$P(h(x), h(x)) \ \lor \ P(h(y), h(b)) \qquad (3)$$

$$\neg P(f(x), y) \ \lor \ \neg P(x, y) \ \lor \ \neg P(y, g(x)) \qquad (4)$$

$$\neg P(x, y) \ \lor \ \neg P(b, c) \qquad (5)$$

Suppose that the atom ordering $\succ$ is a Knuth-Bendix ordering with weight 1 for all function and predicate symbols and variables and the precedence $P > f > g > h > b > c$.

Part (a)

Which literals in the clauses (1)–(5) are maximal in their clause?

Part (b)

Assume that the selection function *sel* selects the second literal in (5) and no other literals. Compute all $\text{Res}^{\succ}_{sel}$ inferences between the clauses (1)–(5). (Do not compute inferences between derived clauses. Do not compute any inferences that violate the restrictions of the calculus.)

Assignment 3 *(E-Algebras)* (12 points)

Let $\Sigma = (\Omega, \emptyset)$ be a first-order signature, let E be a set of Σ-equations such that for every equation $s \approx s'$ in E neither s nor s' is a variable. For any term $t \in \mathrm{T}_\Sigma(X)$ let $[t]$ denote the congruence class of t w.r.t. E.
Prove or refute: For every variable $x \in X$ we have $[x] = \{x\}$.

Assignment 4 *(Reduction orderings)* (12 points)

Let $\Sigma = (\Omega, \emptyset)$ be a finite signature. For $t \in \mathrm{T}_\Sigma(X)$ we define $\mathrm{depth}(t) = \max\{\, |p| \mid p \in \mathrm{pos}(t)\,\}$. The binary relation $\succ$ on $\mathrm{T}_\Sigma(X)$ is defined by:

$s \succ t$ if and only if

$$\#(x, s) \geq \#(x, t) \text{ for all variables } x \text{ and } \mathrm{depth}(s) > \mathrm{depth}(t).$$

Show that $\succ$ is not a reduction ordering.

Assignment 5 *(Critical pairs, Termination)* (8 + 10 = 18 points)

Let $\Sigma = (\Omega, \emptyset)$ with $\Omega = \{b/0,\ c/0,\ f/2,\ g/1,\ h/2\}$ and let R be the following rewrite system:

$$f(f(x, c), x) \to h(x, b) \qquad (1)$$

$$f(y, b) \to g(y) \qquad (2)$$

$$h(x, x) \to g(f(x, c)) \qquad (3)$$

Part (a)

Find a Knuth-Bendix ordering $\succ$ such that $\to_R \subseteq \succ$. Specify the weights and precedence of the ordering.

Part (b)

Compute all critical pairs between rules in R and check whether they are joinable in R.

Assignment 6 *(OBDDs)* (12 points)

Let F_n be a propositional formula over $\{P_1, \ldots, P_n\}$ such that $\mathcal{A}(F_n) = 1$ if and only if $\mathcal{A}$ maps exactly one of the propositional variables $P_1, \ldots, P_n$ to 1 and the others to 0. How many nodes does a reduced OBDD for F_n have (including the leaf nodes $\boxed{0}$ and $\boxed{1}$)?