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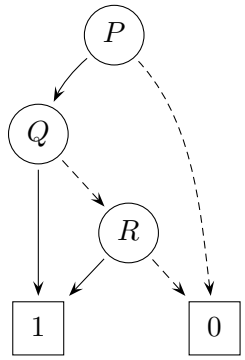
**Uwe Waldmann** **November 20, 2019**

**Tutorials for "Automated Reasoning"**
**Exercise sheet 5**

**Exercise 5.1:** (*2+1+2 P*)

(1) Give a propositional formula $F$ that is represented by the reduced OBDD on the right.




(2) How many different reduced OBDDs over the propositional variables $\{P, Q, R\}$ have exactly one interior (non-leaf) node?

(3) Find a propositional formula $G$ over the propositional variables $\{P, Q, R\}$, such that the reduced OBDD for $G$ has three interior nodes and the reduced OBDD for $F \vee G$ has one interior node. Give the reduced OBDDs for $G$ and $F \vee G$.

**Exercise 5.2:** (*5 P*)

Prove Case 2 of Thm. 2.21 (i. e.: $v$ and $v'$ are non-leaf nodes labelled by the same propositional variable).

**Exercise 5.3:** (*2+3 P*)

Let the signature $\Sigma = (\Omega, \Pi)$ be given by $\Omega = \{+/2, s/1, 0/0\}$ and $\Pi = \emptyset$, and let

$$\begin{aligned} F_1 &= \forall x\,(x + 0 \approx x) \\ F_2 &= \forall x \forall y\,(x + s(y) \approx s(x + y)) \\ F_3 &= \forall x \forall y\,(x + y \approx y + x) \\ F_4 &= \neg\forall x \forall y\,(x + y \approx y + x). \end{aligned}$$

(1) Determine a $\Sigma$-algebra $\mathcal{A}$ with an universe of exactly two elements such that $\mathcal{A}$ is a model of $F_1$, $F_2$, $F_3$.

(2) Determine a $\Sigma$-algebra $\mathcal{A}$ with an universe of exactly two elements such that $\mathcal{A}$ is a model of $F_1$, $F_2$, $F_4$.

**Exercise 5.4:** (*5 P*)
Prove Prop. 3.5: For any $\Sigma$-formula $F$, $\mathcal{A}(\beta)(F\sigma) = \mathcal{A}(\beta \circ \sigma)(F)$.
(It is sufficient if you prove the property for atomic formulas $P(s_1, \ldots, s_n)$, disjunctions $F \vee G$, and universally quantified formulas $\forall x\, F$; the other cases are proved similarly.)

Submit your solution in lecture hall E1.3, Room 001 during the lecture on November 27. Please write the time of your tutorial group (Mon or Tue) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.