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Tutorials for "Automated Reasoning"
Exercise sheet 4

Exercise 4.1: (*7 P*)
Let N be the following set of propositional clauses:

(1) $P \lor \neg Q \lor R$
(2) $P \lor \neg T \lor \neg U \lor V$
(3) $P \lor \neg Q \lor T \lor U \lor \neg V$
(4) $\neg P \lor Q$
(5) $R \lor T$
(6) $R \lor \neg U$
(7) $\neg P \lor S \lor \neg U \lor \neg V$
(8) $\neg R \lor S$
(9) $\neg R \lor T \lor V$
(10) $\neg S \lor T \lor U \lor \neg V$
(11) $\neg T \lor U$
(12) $\neg S \lor \neg T \lor \neg U \lor V$
(13) $\neg U \lor \neg V$

Use the CDCL procedure to check whether N is satisfiable or not; if it is satisfiable, give a model. Use the CDCL inference rules with a reasonable strategy (i.e., use *Fail* or *Backjump* if possible, otherwise use *Unit Propagate* if possible, otherwise use *Decide*). If you use the *Decide* rule, use the largest undefined positive literal according to the ordering $P > Q > R > S > T > U > V$. If you use the *Backjump* rule, determine a suitable backjump clause using the 1UIP method and use the best possible successor state for that backjump clause.

Exercise 4.2: (*3 P*)
The "Purity deletion" rule explained in the *Inprocessing* section is subsumed by other inprocessing rules. By which one(s)? Why?

Exercise 4.3: (*5 P*)
Prove that the "RAT elimination" rule explained in the *Inprocessing* section is satisfiability-preserving:

C is called an *asymmetric tautology* w. r. t. N, if its negation can be refuted by unit propagation using clauses in N.

We say that C has the *RAT property* w. r. t. N, if it is an asymmetric tautology w. r. t. N, or if there is a literal L in C such that $C = C' \vee L$ and all clauses $D' \vee C'$ for $D' \vee \overline{L} \in N$ are asymmetric tautologies w. r. t. N.

Assume that C has the RAT property w. r. t. N. Show that $N \cup \{C\}$ is satisfiable if and only if N is satisfiable.

Exercise 4.4: (*5 P*)
A friend asks you to proofread his bachelor thesis. On page 14 of the thesis, your friend writes the following:

> **Definition 11.** Let N be a set of propositional formulas. The set $\mathit{poscomb}(N)$ of positive combinations of formulas in N is defined inductively by
> (1) $N \subseteq \mathit{poscomb}(N)$;
> (2) if $F, F' \in \mathit{poscomb}(N)$, then $F \vee F' \in \mathit{poscomb}(N)$; and
> (3) if $F, F' \in \mathit{poscomb}(N)$, then $F \wedge F' \in \mathit{poscomb}(N)$.
>
> **Lemma 12.** If N is a satisfiable set of formulas, then every positive combination of formulas in N is satisfiable.
>
> **Proof.** The proof proceeds by induction over the formula structure. Let $G \in \mathit{poscomb}(N)$. If $G \in N$, then it is obviously satisfiable, since N is satisfiable. Otherwise, G must be a disjunction or a conjunction of formulas in $\mathit{poscomb}(N)$. If G is a disjunction $F \vee F'$ with $F, F' \in \mathit{poscomb}(N)$, we know by the induction hypothesis that F is satisfiable. So F has a model. Since this is also a model of $G = F \vee F'$, the formula G is satisfiable. Analogously, if G is a conjunction $F \wedge F'$, with $F, F' \in \mathit{poscomb}(N)$, then both F and F' are satisfiable by induction, so $G = F \wedge F'$ is satisfiable as well.

(1) Is the "proof" correct (yes/no)?

(2) If the "proof" is not correct:

   (a) Which step is incorrect?

   (b) Does the "lemma" hold? If yes, give a correct proof, otherwise give a counterexample.

Submit your solution in lecture hall E1.3, Room 001 during the lecture on November 20. Please write the time of your tutorial group (Mon or Tue) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.