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Tutorials for “Automated Reasoning”
Exercise sheet 2

Exercise 2.1: (4 P)

Which of the following propositional formulas are valid? Which are satisfiable? Which are unsatisfiable?

- (1) $\neg P$
- (2) $P \rightarrow \perp$
- (3) $\perp \rightarrow P$
- (4) $(P \vee Q) \rightarrow P$
- (5) $P \rightarrow (Q \rightarrow P)$
- (6) $Q \rightarrow \neg Q$
- (7) $Q \wedge \neg Q$
- (8) $\neg(\neg P \wedge \neg\neg P)$

Exercise 2.2: (5 P)

Let F , G , and H be propositional formulas. Prove or refute:

- (1) If $F \models G$ and $G \models H$, then $F \models H$.
- (2) If F is satisfiable and G is satisfiable, then $F \wedge G$ is satisfiable.
- (3) If F is satisfiable and $F \rightarrow G$ is satisfiable, then G is satisfiable.
- (4) If $F \vee G$ is valid, then F is valid or G is satisfiable.
- (5) If $F \vee G[F]_p$ is valid, then $F \vee G[\perp]_p$ is valid.

Exercise 2.3: (2+3 P)

Let F be the formula $(Q \rightarrow P) \rightarrow (\neg P \wedge Q \wedge R)$.

- (1) Convert F into an equivalent CNF formula as described in Prop. 2.12.
- (2) Replace the subformulas $Q \rightarrow P$ and $\neg P \wedge Q \wedge R$ by new variables, add the polarity-dependent definitions for the new variables and convert again into a CNF formula.

Exercise 2.4: (6 P)

Prove Proposition 2.14: Let \mathcal{A} be a valuation, let F and G be formulas, and let $H = H[F]_p$ be a formula in which F occurs as a subformula at position p .

If $\text{pol}(H, p) = 1$ and $\mathcal{A}(F) \leq \mathcal{A}(G)$, then $\mathcal{A}(H[F]_p) \leq \mathcal{A}(H[G]_p)$.

If $\text{pol}(H, p) = -1$ and $\mathcal{A}(F) \geq \mathcal{A}(G)$, then $\mathcal{A}(H[F]_p) \leq \mathcal{A}(H[G]_p)$.

(It is sufficient if you consider the boolean connectives \wedge and \neg ; the other cases are proved analogously.)

There will be no lecture on November 6. Submit your solution into the box in front of Room 638 in Bldg E1.5 (MPI-SWS) until November 6.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.