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Tutorials for “Automated Reasoning”
Exercise sheet 13

Exercise 13.1:

Apply the Knuth-Bendix procedure to the set of equations

$$f(0, f(x, f(y, z))) \approx f(f(0, x), f(y, z)) \quad (1)$$

$$f(0, x) \approx 0 \quad (2)$$

$$f(x, 1) \approx x \quad (3)$$

and transform it into a finite convergent term rewrite system; use the Knuth-Bendix ordering with weight 1 for all function symbols and variables and the precedence $f > 1 > 0$. Start by orienting the first equation.

Exercise 13.2:

If the set R in the Knuth-Bendix completion procedure contains two rules whose left-hand sides are equal up to variable renaming, then none of them can be simplified using the other one. In practice, such a situation should never occur. Why?

Exercise 13.3:

Let $\Sigma = (\Omega, \emptyset)$ with $\Omega = \{b/0, f/1, g/1\}$. Which ground terms are in T_∞ for the following TRS?

$$f(f(b)) \rightarrow g(b) \quad (1)$$

$$g(x) \rightarrow g(f(x)) \quad (2)$$

Exercise 13.4:

[Note: Part (c) relies on techniques that will be discussed in the lecture on February 3.]

Let $\Sigma = (\Omega, \emptyset)$ with $\Omega = \{f/2, g/2, h/1, k/1, b/0\}$.

(a) Compute the dependency pairs of the following rewrite system R over Σ :

$$f(x, h(x)) \rightarrow h(k(x)) \quad (1)$$

$$f(h(x), y) \rightarrow g(x, g(h(x), x)) \quad (2)$$

$$g(x, x) \rightarrow f(x, x) \quad (3)$$

$$g(x, y) \rightarrow y \quad (4)$$

$$h(b) \rightarrow b \quad (5)$$

(b) Compute the approximated dependency graph for R (using cap and ren).

(c) Use the subterm criterion to show that R is terminating. If a graph is modified, depict both the original and the modified graph and indicate the strongly connected components in the graphs.

(d) The approximated dependency graph contains an edge from a dependency pair generated by rule (3) to a dependency pair generated by rule (1). Is this edge also contained in the exact dependency graph? Give an explanation.

Bring your solution to the Q&A session on February 5. By lack of time, it will *not* be checked by the tutors.