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Tutorials for “Automated Reasoning”

Exercise sheet 12

Exercise 12.1: (5 P)

Prove Thm. 4.30: If the precedence \succ is total, then the lexicographic path ordering \succ_{lpo} is total on ground terms, i.e., for all $s, t \in T_{\Sigma}(\emptyset)$: $s \succ_{\text{lpo}} t \vee t \succ_{\text{lpo}} s \vee s = t$.

Exercise 12.2: (3+3 P)

Let $\Sigma = (\{f/1, g/2, h/1, b/0, c/0\}, \emptyset)$ and let

$$t_1 = g(h(x), h(c)),$$

$$t_2 = g(x, h(h(x))),$$

$$t_3 = h(g(x, b)),$$

$$t_4 = f(g(x, y)).$$

Determine for each $1 \leq i < j \leq 4$ whether t_i and t_j are incomparable or comparable (and if so, which term is larger) with respect to

- (a) a lexicographic path ordering with precedence $f > g > h > b > c$,
- (b) a Knuth–Bendix ordering with precedence $h > f > g > b > c$, where h has weight 0 and all other symbols have weight 1,

Exercise 12.3: (2+2 P)

- Find a lexicographic path ordering \succ such that $h(h(x)) \succ f(x)$ and $f(g(h(x), y)) \succ h(g(x, f(y)))$.
- Find a Knuth–Bendix ordering \succ such that the set of clauses

$$P(f(x, y), y) \vee P(g(y), g(x)) \quad (1)$$

$$\neg P(f(b, y), y) \vee \neg Q(y, y) \quad (2)$$

$$Q(g(x), g(y)) \vee \neg Q(x, h(y)) \quad (3)$$

is saturated under Res_{sel}^{\succ} , where sel does not select any literals.

Exercise 12.4: (5 P)

Apply the Knuth–Bendix procedure to the set of equations

$$f(f(x)) \approx g(x) \quad (1)$$

$$f(b) \approx c \quad (2)$$

and transform it into a finite convergent term rewrite system; use the Knuth–Bendix ordering with weight 1 for all function symbols and variables and the precedence $g > f > b > c$.

Submit your solution in lecture hall E1.3, Room 001 during the lecture on January 29. Please write the time of your tutorial group (Mon or Tue) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.