



Uwe Waldmann

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Tutorials for “Automated Reasoning”
Exercise sheet 2

Exercise 2.1: (3 P)

Which of the following propositional formulas are valid? Which are satisfiable? Which are unsatisfiable?

- (1) $\neg P$
- (2) $(P \vee Q) \rightarrow P$
- (3) $P \rightarrow (Q \rightarrow P)$
- (4) $Q \rightarrow \neg Q$
- (5) $Q \wedge \neg Q$
- (6) $\neg(\neg P \vee \neg\neg P)$

Exercise 2.2: (2 P)

Let $\Pi = \{P, Q, R\}$. How many models does the Π -formula $(P \wedge Q) \vee (P \wedge \neg R)$ have?

Exercise 2.3: (5 P)

Let F , G and H be propositional formulas. Prove or refute:

- (1) If $F \models G$ and $G \models H$, then $F \models H$.
- (2) If F is satisfiable and G is satisfiable, then $F \wedge G$ is satisfiable.
- (3) If F is satisfiable and $F \rightarrow G$ is satisfiable, then G is satisfiable.
- (4) If $F \vee G$ is valid, then F is valid or G is satisfiable.
- (5) If $F \vee G[F]_p$ is valid, then $F \vee G[\perp]_p$ is valid.

Exercise 2.4: (4 P)

Let F be the formula $(Q \rightarrow P) \rightarrow (\neg P \wedge Q \wedge R)$.

- (1) Convert F into an equivalent CNF formula as described in Prop. 2.12.
- (2) Replace the subformulas $Q \rightarrow P$ and $\neg P \wedge Q \wedge R$ by new variables, add the polarity-dependent definitions for the new variables and convert again into a CNF formula.

Exercise 2.5: (5 P)

Prove Proposition 2.14: Let \mathcal{A} be a valuation, let F and G be formulas, and let $H = H[F]_p$ be a formula in which F occurs as a subformula at position p .

If $\text{pol}(H, p) = 1$ and $\mathcal{A}(F) \leq \mathcal{A}(G)$, then $\mathcal{A}(H[F]_p) \leq \mathcal{A}(H[G]_p)$.

If $\text{pol}(H, p) = -1$ and $\mathcal{A}(F) \geq \mathcal{A}(G)$, then $\mathcal{A}(H[F]_p) \leq \mathcal{A}(H[G]_p)$.

(It is sufficient if you consider the boolean connectives \wedge and \neg ; the other cases are proved analogously.)

Submit your solution in lecture hall E1.3, Room 003 during the lecture on November 9. Please write your name and the time of your tutorial group (Tue 8–10, Wed 8–10, or Wed 14–16) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.