Assignment 1 *(DPLL)* (10 points)

Let N be some set of propositional clauses over $\Pi = \{P, Q, R, S, T, U, V\}$. Suppose that we use the relation $\Rightarrow_{\text{DPLL}}$ to test whether N is satisfiable or not, and that, during the DPLL-derivation, we reach the state

$$P^d\ Q^d\ \neg R\ S^d\ T^d\ \neg U\ V \parallel N.$$

Assume that the clauses C_1, C_2, C_3, C_4 are logical consequences of N:

$$C_1 = \neg P \vee R \vee \neg T \qquad C_3 = \neg P \vee \neg Q \vee \neg S \vee \neg T$$
$$C_2 = \neg P \vee \neg T \vee U \qquad C_4 = \neg P \vee Q \vee R$$

Determine for each of the clauses C_1, C_2, C_3, C_4 whether it satisfies the requirements for a backjump clause. If it does, determine additionally the *best possible* successor state for this backjump clause. Which of the clauses C_1, C_2, C_3, C_4 should one use in practice as a backjump clause?

Assignment 2 *(Resolution)* (12 + 4 = 16 points)

Part (a)

Let N be the set of first-order clauses $\{C_1, C_2, C_3, C_4\}$ over the signature $\Sigma = (\{b/0,\ c/0,\ d/0,\ f/1,\ g/1\},\ \{P/2,\ Q/2\})$:

$$\begin{aligned}
C_1 &= P(x,x)\ \vee\ P(f(x'), f(f(x')))\\
C_2 &= P(b,y)\ \vee\ \neg Q(y,y')\\
C_3 &= \neg P(c,z)\ \vee\ Q(g(z),z)\\
C_4 &= \neg P(d,z')
\end{aligned}$$

Compute $Res^*(N)$. State for each derived clause from which premise(s) it is derived. (You need not write down the side computations for the mgu's.)

Part (b)

What can one say about the (un-)satisfiability of N? Give a brief explanation.

Assignment 3 *(Resolution)* (10 + 4 = 14 points)

Part (a)

Let N be a set of (not necessarily ground) first-order clauses. Let $D = \neg A$ be a negative unit clause such that no resolution inference between any clause $C \in N$ and D is possible. Prove that no resolution inference between any clause $C' \in Res^*(N)$ and D is possible.

Part (b)

Does the property also hold if D is a positive unit clause or an arbitrary clause? Give a brief explanation.

Assignment 4 *(Multisets, Clause Orderings)* (10 points)

Find a total ordering $\succ$ on the atoms $P(b), P(c), Q(b), Q(c)$ such that the following properties hold for the associated clause ordering $\succ_C$:

$$P(b) \vee \neg Q(c) \succ_C \neg P(b) \vee Q(c) \quad (1)$$
$$P(b) \vee \neg P(c) \succ_C \neg P(c) \vee \neg Q(b) \quad (2)$$
$$\neg P(b) \vee P(c) \vee Q(c) \succ_C Q(c) \vee Q(c) \vee Q(c) \quad (3)$$

Assignment 5 *(Herbrand Interpretations)* (4 + 8 + 6 = 18 points)

Let $\Sigma = (\Omega, \Pi)$ be a first-order signature with $\Omega = \{b/0, c/0, d/0\}$ and $\Pi = \{P/1, Q/1\}$. Let F be the Σ-formula $\neg P(b) \wedge Q(c) \wedge Q(d)$.

Part (a)

How many different Herbrand models over Σ does F have?

Part (b)

State for each of the following Σ-formulas $G_1, G_2, G_3, G_4, G_5, G_6$ whether they hold in *all* Herbrand models of F, *some, but not all* Herbrand models of F, or *none* of the Herbrand models of F:

$$G_1 = \exists x\, P(x) \qquad G_3 = \exists x\, Q(x) \qquad G_5 = \forall x\, (P(x) \rightarrow Q(x))$$
$$G_2 = \forall x\, P(x) \qquad G_4 = \forall x\, Q(x) \qquad G_6 = (\exists x\, P(x)) \rightarrow (\forall x\, Q(x))$$

(Note on grading: You do not have to give explanations. However, you need at least three correct answers to get any points for part (b). Missing answers count like false answers.)

Part (c)

Give a Σ-algebra $\mathcal{A}$ with universe $U_{\mathcal{A}} = \{1, 2\}$, such that $\mathcal{A} \models F$, but $\mathcal{A} \not\models G_5$.

Assignment 6 *(First-order Logic)* (12 points)

Let $\Sigma = (\Omega, \Pi)$ be a first-order signature. Define the signature $\Sigma' = (\Omega, \Pi')$, where $\Pi' = \{\, P/0 \mid P/0 \in \Pi \,\} \cup \{\, Q/1 \mid Q/n \in \Pi,\ n \geq 1 \,\}$. For every Σ-formula F without equality let drop(F) be the Σ'-formula that one obtains from F by replacing every atom $Q(t_1, \dots, t_n)$ in F with $n \geq 1$ by $Q(t_1)$. E.g., if

$$F = \exists y \left(R \wedge \forall x\, (Q(g(x, b)) \vee S(y, x, f(y)))\right),$$

then

$$\text{drop}(F) = \exists y \left(R \wedge \forall x\, (Q(g(x, b)) \vee S(y))\right).$$

Prove: If drop(F) is satisfiable, then F is satisfiable. (Note: Somewhere in the proof you need an induction over the structure of formulas. It is sufficient if you check the base cases and $\wedge$, $\neg$, and $\exists$. The other boolean connectives and quantifiers can be handled analogously; you may omit them.)