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Tutorials for "Automated Reasoning"
Exercise sheet 6

Exercise 6.1: *(3 P)*
Prove the following statement: If N is a set of propositional formulas and C is a propositional formula such that $N \models C$, then there exists a finite subset $M \subseteq N$ such that $M \models C$.

Exercise 6.2: *(5 P)*
Compute $\mathit{Res}(N)$ for the following set N of (implicitly universally quantified) clauses:

$$
\begin{array}{ll}
(1) & P(x,x) \lor P(h(x',b), h(c,x'')) \\
(2) & \neg P(y, f(y)) \lor Q(g(y)) \\
(3) & \neg Q(z) \lor P(d,z)
\end{array}
$$

(You may write down the side computations for the mgu's, but you don't have to. Do not compute $\mathit{Res}^*(N)$!)

Exercise 6.3: *(4 P)*
Give a (preferably simple) example of a clause C such that $\mathit{Res}^*(\{C\})$ is infinite.

Exercise 6.4: *(6 P)*
Prove that the resolution rule for general clauses

$$\frac{D \lor B \qquad C \lor \neg A}{(D \lor C)\sigma} \quad \text{if } \sigma = \mathrm{mgu(A,B)}$$

is sound, that is,

$$\{\forall x_1, \ldots, x_k(D \lor B), \forall y_1, \ldots, y_n(C \lor \neg A)\} \models \forall z_1, \ldots, z_m((D \lor C)\sigma)$$

Challenge Problem: (*6 Bonus Points*)
Prove part (ii) of Prop. 3.23: If $\sigma \le \tau$ and $\tau \le \sigma$, then there exist variable renamings δ and δ' (i.e., *bijective* substitutions mapping variables to variables), so that $x\sigma\delta = x\tau$ and $x\tau\delta' = x\sigma$ for every x in X.

Submit your solution during the tutorial on November 26 or 27 or in lecture hall E1.3, Room 001 during the lecture on November 27. Please write your name and the date of your tutorial group (Tue, Wed) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.