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Tutorials for “Automated Reasoning”
Exercise sheet 5

Exercise 5.1: (4 P)

Transform the following formula into an equisatisfiable set of universally quantified clauses using the method described in Sect. 3.6:

$$\forall x \forall y \exists z (P(x, z) \wedge \neg \forall x \exists z Q(x, y, z))$$

Exercise 5.2: (4+2 P)

Let N be the following set of ground clauses:

$$\{ P \vee Q, P \vee \neg Q, \neg P \vee Q, \neg P \vee \neg Q \}$$

(i) Show that $N \vdash_{Res} \perp$, that is, derive \perp from N using the Resolution and the Positive Factorization rule.

(ii) Why is it impossible to derive the empty clause from these clauses without using factorization?

Exercise 5.3: (4+4 P)

Let N be the set containing the following ground clauses:

$$\begin{aligned} C_1 &= P(a) \vee \neg Q(a) \vee \neg Q(b) \\ C_2 &= P(a) \vee P(a) \vee Q(b) \\ C_3 &= P(b) \vee Q(a) \vee Q(b) \\ C_4 &= Q(a) \vee Q(b) \vee Q(b) \\ C_5 &= \neg P(a) \vee Q(b) \\ C_6 &= \neg P(b) \end{aligned}$$

Let the ordering on ground atoms be given by $P(a) \succ P(b) \succ Q(a) \succ Q(b)$.

(i) Order the clauses in N according to the associated clause ordering \succ_C .

(ii) Compute the candidate interpretation I_N^\succ . Which clauses are productive, what do they produce, which clause is the minimal counterexample (if it exists)?

Challenge Problem: (5 Bonus Points)

A Σ -algebra \mathcal{A} is called *term-generated*, if for every $a \in U_{\mathcal{A}}$ there is a *ground* term $t \in T_{\Sigma}$ such that $a = \mathcal{A}(\beta)(t)$. (Note that, if t is ground, then $\mathcal{A}(\beta)(t) = \mathcal{A}(\beta')(t)$ for all assignments β and β' .) Prove that a closed prenex formula without existential quantifiers has a model if and only if it has a term-generated model.

Submit your solution during the tutorial on November 19 or 20 or in lecture hall E1.3, Room 001 during the lecture on November 20. Please write your name and the date of your tutorial group (Tue, Wed) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.