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Tutorials for "Automated Reasoning"
Exercise sheet 3

Exercise 3.1: (*4 P*)
Let F be the formula $(Q \rightarrow P) \rightarrow (\neg P \wedge Q \wedge R)$.

(1) Convert F into an equivalent CNF formula as described in Prop. 2.9.

(2) Replace the subformula $\neg P \wedge Q \wedge R$ by a new variable S, add the polarity-dependent definition for S, and convert again into a CNF formula.

Exercise 3.2: (*5 P*)
Let N be the following set of propositional clauses:

$$\begin{array}{ccccccccccccc}
\neg P & & & \vee & \neg R & & & \vee & \neg T & & & & (1) \\
\neg P & & & & & & & \vee & T & \vee & \neg U & & (2) \\
 & & & & \neg R & & & \vee & T & \vee & U & & (3) \\
 & & \neg Q & \vee & \neg R & \vee & S & & & & & & (4) \\
\neg P & & & \vee & R & \vee & \neg S & & & & & & (5) \\
 & & Q & & & & & & & \vee & \neg U & & (6) \\
P & & & & & & & & & \vee & U & & (7) \\
P & \vee & \neg Q & & & & & & & \vee & \neg U & & (8)
\end{array}$$

Assume that during a DPLL-derivation, we have reached the configuration $P^d Q^d R^d S \neg T U \parallel N$. Give two different backjump clauses that can be used in this situation and give the successor state with respect to $\Rightarrow_{\mathsf{DPLL}}$ for each of these backjump clauses.

Exercise 3.3: (*4 P*)
Let F and G be propositional formulas. Prove or refute:

(1) If F is satisfiable or G is satisfiable, then $F \vee G$ is satisfiable.

(2) If F is satisfiable and G is satisfiable, then $F \wedge G$ is satisfiable.

Exercise 3.4: (*5 P*)
Let P be a propositional variable. For any formula F let $\text{negate}_P(F)$ be the result of replacing every occurrence of P in F by $\neg P$. (For instance, $\text{negate}_P(\neg P \rightarrow (P \vee Q))$ is $\neg\neg P \rightarrow (\neg P \vee Q)$.) Prove: If the formula F is satisfiable, then $\text{negate}_P(F)$ is satisfiable.

Challenge Problem: (*5 Bonus Points*)
Show that the following property holds: Let $\rightarrow_1$ and $\rightarrow_2$ be two binary relations over a non-empty set A, such that $(\rightarrow_1 \cup \rightarrow_2)$ is transitive. Then $(\rightarrow_1 \cup \rightarrow_2)$ is terminating if and only if $\rightarrow_1$ and $\rightarrow_2$ are terminating. (Hint: Start with the assumption that there is an infinite $(\rightarrow_1 \cup \rightarrow_2)$ chain.)

Submit your solution during the tutorial on November 5 or 6 or in lecture hall E1.3, Room 001 during the lecture on November 6. Please write your name and the date of your tutorial group (Tue, Wed) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.